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THE MAUTNER PHENOMENON FOR GENERAL UNITARY REPRESENTATIONS

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Dedicated to Gerhard Hochschild on the occasion of his 65th birthday

The 'Mautner phenomenon' for unitary representations is a general assertion of the form that if $x(t)$ is a one parameter subgroup of a Lie group G, π a unitary representation of G on a Hilbert space $\mathscr{H} = \mathscr{H}(\pi)$ and v a vector in $\mathscr{H}$ which is fixed by $x(t)$; i.e., $\pi(x(t))v = v$, then v must also be fixed by a generally much larger subgroup H of G. How much larger H is than the original one parameter group depends in a general way on how noncommutative the group G is. Our purpose here is to establish a very general result of this nature which we believe to be the best possible result of this kind.

1. The 'Mautner phenomenon' for unitary representations is a general assertion of the form that if $x(t)$ is a one parameter subgroup of a Lie group G, π a unitary representation of G on a Hilbert space $\mathscr{H} = \mathscr{H}(\pi)$ and v a vector in $\mathscr{H}$ which is fixed by $x(t)$; i.e., $\pi(x(t))v = v$, there v must also be fixed by a generally much larger subgroup H of G. How much larger H is than the original one parameter group depends in a general way on how noncommutative the group G is. Our purpose here is to establish a very general result of this nature which we believe to be the best possible result of this kind. The first appearance of such results was in [12] clarifying earlier work in [7], and this theme has appeared in a number of subsequent papers, notably [1], [2], [13], [14], [3], [8], [6], and [15]. The initial application of such results was to the ergodic theory of homogeneous flows, and as can be seen from the other works cited, these applications to ergodic theory have remained central. More recently such results have found applications in the general area of automorphic forms on reductive groups, and more generally the results embodied in the general Mautner phenomenon reflect an interesting and it seems significant property of unitary representations of general Lie groups. In a subsequent papar [5] it will be shown how to use the results here to obtain very comprehensive results on the ergodic theory of homogeneous flows.

Let us now formulate the main result. If G is a connected Lie group, $\mathfrak{g}$ its Lie algebra and $X \in \mathfrak{g}$, we shall say that X (or the corresponding one parameter subgroup $x(t) = \exp(t(X))$ is *ad-compact* if $\mathrm{Ad}(x(t))$ acting linearly on $\mathfrak{g}$ is contained in a compact subgroup of the full automorphism group of $\mathfrak{g}$. This is clearly equivalent to