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ANALYTIC SUBGROUPS OF AFFINE ALGEBRAIC GROUPS, II

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Dedicated to Gerhard Hochschild on the occasion of his 65th birthday

Let H be Zariski-dense analytic subgroup of the connected linear complex algebraic group G. It is known that there is a torus T in G with $G = HT$ and $H \cap T$ discrete in H. This paper gives equivalent conditions for $H \cap T$ to be trivial, and considers the connection between these conditions and left algebraic group structures on H induced from the coordinate ring of G.

Let G be a connected linear complex algebraic group, and let H be a Zariski-dense analytic subgroup of G which is integral in the sense of [2, Defn. 1, p. 386]. In [10, Thm. 3] it was shown that there exists an algebraic torus T in G with $G = HT$ such that the Lie algebra of T is a vector space complement to the Lie algebra of H in the Lie algebra of G; T is called a *complementary torus* to H in G. The principal results of this paper deal with conditions under which such a complementary torus meets H trivially. The existence of such a torus is connected, by [10, Prop. 6] and [10, Prop. 7], to left algebraic group structures on H in the sense of [8, Defn. 2.1].

We recall some terminology: let H be an analytic group, let f be an analytic function on H, and let x be in H. Then $x \cdot f$ (respectively, $f \cdot x$) is the function on H whose value at y is $f(yx)$ (respectively, $f(xy)$). f is *representative* if $\{x \cdot f \mid x \in H\}$ spans a finite-dimensional vector space, and $R(H)$ denotes the Hopf algebra of all representative functions on H [5]. A representative function f on H is *semi-simple* if the representation of H on the span of $\{x \cdot f \mid x \in H\}$ is semi-simple, and $R(H)_s$ denotes the subalgebra of all semi-simple representative functions on H [5]. An *analytic left algebraic group structure* on H is a finite-type C-subalgebra A on $R(H)$ such that (1) if $f \in A$ and $x \in H$, $f \cdot x \in A$, and (2) evaluations at element of H correspond bijectively to C-algebra maps from A to C [8, Defn. 2.1]. A *nucleus* of H is a closed, solvable, simply connected normal subgroup K such that H/K is reductive [6, p. 112]. An *additive character* of H is a homomorphism from H to the additive analytic group C and $X^+(H)$ is the free abelian group of additive characters of H. H is an *FR* group if H has a faithful finite-dimensional representation; if V is the space of such a representation then H is a Zariski-dense analytic subgroup of the Zariski-closure of H in $\mathrm{GL}(V)$ which is an algebraic group.