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SUPERALGEBRAS

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Dedicated to Gerhard Hochschild on the occasion of his 65th birthday

1. Introduction. The theory of graded Lie algebras, now more widely called Lie superalgebras, underwent a very rapid development starting about 1973, inspired by the interest expressed in the sudject by physicists. I was active in the field for about a year, during 1975 and 1976. Thus far I have published only the announcement [16] (jointly with Peter Freund of Chicago's Physics Department, to whom I am enormously indebted); in addition, the summary [29] is to appear.

The present mature state of the field, and the fact that Hochschild (partly in collaboration with Djoković) made several important contributions, make this an appropriate occasion to publish some further details. Although Victor Kac has brilliantly solved the main problems, there remains the possibility that the different methods I used retain some independent interest.

The large bibliography is intended to be complete on mathematical references not contained in [9]; there is also a selection of physics papers. I hope this bibliography will be useful to some readers.

This article is written so as to keep the overlap with [29] to a minimum.

2. Invariant forms. When I began studying Lie superalgebras I imitated [46] and selected as an initial goal the classification of those simple Lie superalgebras (over an algebraically closed field of characteristic 0) that admit a suitable invariant form.

For basic definitions and facts about Lie superalgebras, I refer to [25]. I shall just recall that if ϕ is a superrepresentation of the Lie superalgebra L then

$$(x, y) = \mathrm{STr}\,(\phi(x)\phi(y))$$

is an invariant form on L, where STr denotes the supertrace. This can be extended to "projective representation", following the model of [28, p. 66], but since the setup will shortly be axiomatic anyway, I shall not pursue the details here.

Assume now that the form ψ on L induced by ϕ is nondegenerate. Write $L = L_0 + L_1$, with L_0 and L_1 the even and odd parts of L. We have that ψ is symmetric on L_0, skew on L_1, and that L_0 and L_1 are orthogonal relative to ψ. It follows that ψ remains non-