PACIFIC JOURNAL OF MATHEMATICS
Vol. 86, No. 1, 1980

A COHOMOLOGICAL INTERPRETATION OF BRAUER GROUPS OF RINGS

RAYMOND T. HOOBLER

Dedicated to Gerhard Hochschild on the occasion of his 65th birthday

Quillen's proof of the Serre conjecture introduced a new tool for passing from local to global results on affine schemes. We use this to prove the theorem below characterizing the image of the injection $i\colon Br(X) \to H^2(X_{et}, G_m)$ when $X = \operatorname{Spec} A$, is a regular scheme. A result of M. Artin then allows us to conclude that $Br(X) \cong H^2(X_{et}, G_m)$ if $X = \operatorname{Spec} A$ is a smooth, affine scheme over a field. For such rings, this proves the Auslander Goldman conjecture [2], $Br(A) = \cap Br(A_\mathfrak{p})$, $\mathfrak{p} \in P(A)$, the set of height one primes of A.

We begin with following theorem.

THEOREM. *Let $X = \operatorname{Spec} A$ be a regular scheme. If $c \in H^2(X_{et}, G_m)$ and $c_y = i([\Lambda_y])$ in $H^2(\operatorname{Spec}(A_{\mathfrak{m}_y})_{et}, G_m)$ for all closed points $y \in X$, then $c = i([\Lambda])$.*

Proof. If $f \in A$, let c_f denote the restriction of c to

$$H^2(\operatorname{Spec}(A_f)_{et}, G_m) .$$

Let $S = \{f \in A \mid c_f = i([\Lambda])$ for some Azumaya algebra Λ over $A_f\}$. We will show that S is an ideal. Then $S = A$ since the hypothesis on c prevents S from being contained in any maximal ideal of A.

Suppose $f_1, f_2 \in S$ and $f \in Af_1 + Af_2$. Then $\operatorname{Spec}(A_f) = D_{f_1} \cup D_{f_2}$ where $D_{f_i} = \operatorname{Spec}(A_{ff_i})$. Hence we may assume $A_f = A$ and $\operatorname{Spec}(A)$ is covered by $D_{f_1} \cup D_{f_2}$. Let Λ_1, Λ_2 be Azumaya algebras over A_{f_1}, A_{f_2} with $i([\Lambda_1]) = c_{f_1}$ and $i([\Lambda_2]) = c_{f_2}$. Since i is injective, $[\Lambda_{1f_2}] = [\Lambda_{2f_1}]$; that is, there are locally free coherent $A_{f_1f_2}$ modules P_1, P_2 such that $\Lambda_{1f_2} \otimes \operatorname{End}(P_1) \cong \Lambda_{2f_2} \otimes \operatorname{End}(P_2)$. Since $K^0(A_{f_i}) \to K^0(A_{f_1f_2})$ is onto (A is regular) [3] and we may assume the rank of P_i is large, there are locally free coherent A_{f_i} modules Q_i such that $Q_{if_i} \cong P_i$ [3, Chapter IX, 4.1]. Replacing Λ_i by $\Lambda_i \otimes \operatorname{End}(Q_i)$, we may assume that $\Lambda_{1f_2} \cong \Lambda_{2f_1}$. Using this patching isomorphism we produce an algebra Λ with $\Lambda_{f_1} \cong \Lambda_1$, $\Lambda_{f_2} \cong \Lambda_2$. Since $H^2(X_{et}, G_m) \to H^2(D_{f_iet}, G_m)$ is a monomorphism, $c = i([\Lambda])$.

COROLLARY 1. *Let $X = \operatorname{Spec}(A)$ be a smooth k scheme where k is a field. Then $Br(X) \cong H^2(X_{et}, G_m)$.*

Proof. Since X is regular, $H^2(X_{et}, G_m)$ is torsion [4]. If $c \in$