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INVARIANTS, MOSTLY OLD ONES

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Dedicated to Gerhard Hochschild on the occasion of his 65th birthday

1. Introduction. Let G be the group with p elements where p is a prime number and let k be a field of characteristic p. Then

$$V_n \cong k[x]/(x-1)^n \quad \text{for } n = 1, 2, \cdots, p$$

are the only indecomposable $k[G]$-modules (observe that $V_p = k[G]$ is free). The rth symmetric power $S^r V_{n+1}$ can be written as a direct sum of indecomposables. Let $b_{n,r}$ denote the number of indecomposables for p large (i.e., $p > nr + 1$) and define the "false" Hilbert series by

$$\psi_n(t) = \sum_{r=0}^{\infty} b_{n,r} t^r .$$

One way to find e.g., $\psi_3(t)$ is to actually compute the decompositions of $S^r V_4$ and counting the components. Then we get the following series for $b_{3,r}$

$$1, 1, 2, 3, 5, 6, 8, 10, 13, 15, 18, 21, 25, 28, \cdots .$$

Guessing a difference equation and solving for $b_{3,r}$ and adding up we get $\psi_3(t)$. For $n = 5$ this method is too tedious and ψ_5 and ψ_6 in [1] were found by other methods (see Ch. V in [1]). After the manuscript of [1] was completed I found that ψ_n for 2, 3, 4 agreed with the generating function for the number of covariants of a binary form of degree n in Faa de Bruno [4]. Later I learned that Franklin and Sylvester a century ago computed ψ_n for $n = 1, 2, \cdots, 10$ and 12 and that our ψ_5 and ψ_6 up to some misprints agreed with theirs.

That this agreement is no coincidence is explained in § 2. It turns out that our G-invariants are identical with what Dickson [3] calls a formal modular semi invariant. For p large they agree with the leading terms (which are semi invariants) of covariants in characteristic zero. Thus from [1] we get the following integral formula for the counting function of covariants

$$\psi_n(t) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1 + \cos\varphi}{\prod_{\nu=0}^{n}(1 - te^{i(n-2\nu)\varphi})} d\varphi .$$

In § 3 it is proved that

$$\psi_n(t^{-1}) = (-1)^n t^{n+1} \psi_n(t)$$