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EXISTENCE OF EIGENVALUES FOR SECOND-ORDER DIFFERENTIAL SYSTEMS

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The paper is concerned with establishing the existence of eigenvalues for the second order differential system $y' = k(x, \lambda)z$, $z' = g(x, \lambda)y$, together with boundary conditions $y(a) = A(\lambda)y(b) + B(\lambda)z(b)$, $z(a) = C(\lambda)y(b) + D(\lambda)z(b)$. A general theorem is obtained establishing the existence of eigenvalues for both self-adjoint and nonself-adjoint boundary problems. This result is then simplified for nonself-adjoint problems, extending the previous work of H. J. Ettlinger and E. Kamke.

1. Introduction. Second-order differential systems involving a parameter, together with boundary conditions at two points, have played a fundamental role in many physical and mechanical processes. The mathematical study of these boundary value problems "began" with the fundamental work of Sturm and Liouville and has flourished ever since.

In this paper, the differential system

$$\begin{aligned} y' &= k(x, \lambda)z , \\ z' &= g(x, \lambda)y , \end{aligned} \tag{1}$$

is considered, where $k(x, \lambda)$ and $g(x, \lambda)$ are real-valued functions on $X: -\infty < a \leqq x \leqq b < \infty$, $L: \lambda_\# - \eta < \lambda < \lambda_\# + \eta$, $0 < \eta \leqq \infty$. The system (1) is studied together with the boundary conditions

$$\alpha_1(\lambda)y(a, \lambda) - \beta_1(\lambda)z(a, \lambda) = \gamma_1(\lambda)y(b, \lambda) - \delta_1(\lambda)z(b, \lambda) , \tag{2a}$$

$$\alpha_2(\lambda)y(a, \lambda) - \beta_2(\lambda)z(a, \lambda) = \gamma_2(\lambda)y(b, \lambda) - \delta_2(\lambda)z(b, \lambda) . \tag{2b}$$

The problem (1, 2a, 2b) has been studied H. J. Ettlinger [3, 4, 5] and E. Kamke [6, 7]. G. D. Birkhoff [1] also studied this problem, but he considered a second-order differential equation rather than a system. However, his equation may be written as a system and the results remain intact.

Under the hypothesis that the boundary problem be self-adjoint and that the coefficient of the differential equation satisfy a monotoniety condition with respect to the parameter, Birkhoff established the existence of an infinite sequence of characteristic values for the boundary problem and determined the oscillatory behavior of the associated solutions.

Later, H. J. Ettlinger, in a series of papers, considered both the self-adjoint and nonself-adjoint boundary problems. By assuming