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PADÉ APPROXIMANTS ON BANACH SPACE OPERATOR EQUATIONS

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We examine an operator equation with a linear compact kernel in a Banach space and the Padé Approximant of its solution under a functional. We give a sufficient condition for convergence of a subsequence of Padé Approximants to the solution.

1. **Introduction.** If one is handling the Padé Approximation technique in multi-particle scattering theory one is interested in convergence. For the two-body case the scattering problem can be formulated as an operator equation with a compact kernel in Hilbert space. Baker [1] proved a result on the convergence of Padé Approximants derived from an operator equation in Hilbert space. Pointwise convergence of a series of Padé Approximants is established for solutions of operator equations as for the two-body scattering partial wave decomposed Lippmann Schwinger kernel and for trace class and compact operators under assumptions on subspace projection sequences. Going over to more particles one usually works in Banach spaces. In the three-particle case Faddeev [4] established an operator equation with compact kernels in a certain Banach space.

Baker's investigation is not easy to generalize onto Banach spaces because of the use of orthogonal projections. Nevertheless we prove a similar result which is mainly based on the properties of cyclic subspaces generated by the inhomogeneity g appearing in the operator equation and the kernel A, which is performed in §2.

In §3 we go over to Hilbert space. Our proposition can be formulated by means of the aperture of two subspaces, as defined by Nagy [8], Krein [6], Krasnoselskii [7]. Finally we discuss cases of validity.

2. **Convergence theorem.** Let us first present definitions. For standard notation used here see [3], [10], [12].

B is a Banach space, B^* its continuous dual space, A a linear compact operator mapping B into B, A^* the adjoint, g is an element of B, h^* an element of B^*. Let λ be a complex number and for $\lambda \neq 0$ let λ^{-1} be an element of $\rho(A)$, the resolvent set of A. The operator equation is

$$f = g + \lambda Af . \tag{2.1}$$

A unique solution exists.