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FINITE HEREDITARY NEAR-RING-SEMIGROUPS

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We generalize the concept of a ring-semigroup to that of a near-ring-semigroup, thus obtaining a much larger class of semigroups. Our main result will be the classification of finite hereditary near-ring-semigroups.

A multiplicative semigroup $(S, \cdot)$ is called a ring-semigroup if addition, $+$, can be defined on S so that $(S, +, \cdot)$ is a ring. It is clear that not every semigroup is a ring-semigroup, thus, one seeks to study ring-semigroups with additional restrictions. Some of the recent activities along this direction are as follows: Ligh classified in [8] all the ring-semigroups in which each subsemigroup containing 0 is also a ring-semigroup. On the opposite end of Ligh's work, H. J. Shyr [12] showed that every subsemigroup of a free semigroup with zero is not a ring-semigroup. Using Ligh's result in [8], the present authors [5] determined all the semigroups that are not ring-semigroups but each proper subsemigroup containing zero is a ring-semigroup.

For a survey of ring-semigroups, see [9].

2. Preliminaries.

DEFINITION 1. A (left) near-ring R is a system with two binary operations, $+$ and $\cdot$, such that (i) $(R, +)$ is a group, (ii) $(R, \cdot)$ is a semigroup, (iii) $x(y + z) = xy + xz$ for all $x, y, z \in R$, and (iv) $0x = 0$ for all $x \in R$.

For basic facts about near-rings, see [10]. Note that (right) near-rings are considered in [10].

DEFINITION 2. Let $(S, \cdot)$ be a multiplicative semigroup. Then S is called a near-ring-semigroup (NR-semigroup) if addition, $+$, can be defined on S so that $(S, +, \cdot)$ is a near-ring. An NR-semigroup is said to be hereditary if every subsemigroup containing 0 is an NR-semigroup.

REMARK 1. Suppose S is a hereditary NR-semigroup and T is a subsemigroup of S. The near-ring T need not be a sub-near-ring of S. The problem of characterizing all the rings $(R, +, \cdot)$ in which each subsemigroup of $(R, \cdot)$ is a subring was begun in [3] and a complete solution is given in [6] and [8]. Motivated by the above problem, Ligh [8] obtained a complete classification of here-