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THE COMPUTATION OF THE GENERALIZED SPECTRUM OF CERTAIN TOEPLITZ OPERATORS

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In an earlier memoir, "Charting the operator terrain," a new generalized spectrum for a bounded operator T on a separable Hilbert space, was defined as follows: Let $C^*(T)$ denote the C^*-algebra generated by T and the identity operator. We say another operator S is weakly contained in T if there exists a *-representation φ of $C^*(T)$ which maps the identity into an identity operator and $\varphi(T)=S$. The "spectrum" of T, denoted $\hat{T}$, is defined to be the space of unitary equivalence classes of irreducible operators weakly contained in T. In this paper this spectrum is explicitly computed for certain specific Toeplitz operators.

The purpose of the memoir [3] was to establish this "spectrum" as a natural generalization of the ordinary (scalar) spectrum of an operator. From the point of view of the theoretical structure the argument is quite convincing. Thus the spectrum is always non-empty and admits a (in general non-Hausdorff) topology relative to which it is compact. If T is normal, $\hat{T}$ may be identified with the ordinary spectrum. Further for a large class of well behaved (smooth) operators one obtains a theory analogous to the ordinary spectral multiplicity theory for normal operators. Thus to each (smooth) operator T one may associate a σ-finite measure class μ on $\hat{T}$ and a multiplicity function f defined on the measures absolutely continuous with respect to μ. These three invariants, $\hat{T}$, μ, f, then determine the operator up to unitary equivalence. This theory reduces to the ordinary spectral multiplicity theory when the operator is normal.

The difficulty with the theory is more practical than theoretical. Since the triplet $(\hat{T}, \mu, f)$ is a complete set of unitary invariants, the complexity of a nonnormal operator is mirrored in the complexity of the spectrum $\hat{T}$. Indeed $\hat{T}$ is a complete algebraic invariant for T in the sense that if S is another operator, then there is a C^*-algebra isomorphism φ of $C^*(T)$ onto $C^*(S)$ such that $\varphi(T) = S$, if and only if $\hat{T} = \hat{S}$. While we feel at home with the ordinary spectrum as a subset of the complex plane, we are somewhat intimidated by this space of equivalence classes of irreducible operators. The purpose of this paper is to make this generalized spectrum a bit less imposing by describing it concretely for certain