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RECURSION FORMULAS FOR THE HOMOLOGY OF $\Omega(X \vee Y)$

G. DULA AND E. KATZ

A recursion formula for $H(\Omega(X \vee Y))$, the homology of the loop space of the wedge of the spaces X and Y is established when ΩX and ΩY are connected, and have finite dimensional homology. The recursion formula is expressed in terms of $H(\Omega X)$ and $H(\Omega Y)$, and applies to dimensions higher than a fixed integer which depends on the dimension of the highest nonvanishing homologies of ΩX and ΩY. A similar but much simpler recursion formula for $H(\Omega X) \amalg H(\Omega Y)$, the co-product of the two algebras $H(\Omega X)$ and $H(\Omega Y)$ is also formulated. If G_1 and G_2 are topological groups and $G_1 * G_2$ is their co-product in the category, then our results definitely hold for $H(G_1 * G_2)$ by replacing ΩX by G_1, ΩY by G_2, and $\Omega(X \vee Y)$ by $G_1 * G_2$.

1. Introduction. Over a field $H(\Omega(X \vee Y))$ equals $H(\Omega X) \amalg H(\Omega Y)$ [1] [2], a fact which substantially simplifies the problem of computing the homology of $\Omega(X \vee Y)$. Over a Dedekind domain a torsion factor is added [5] [3] which significantly complicates the situation. Taking a principal ideal domain as the coefficient ring, $H(\Omega(X \vee Y))$ was computed in [3]. However, even if ΩX and ΩY are finite dimensional, those computations call for an increasing number of manipulations as the dimension of the homology to be computed gets higher. If n_1 and n_2 are the highest dimensions of non vanishing homologies of ΩX, ΩY, then for any $k > 3(n_1 + n_2) + 4$ we introduce a recursion formula which expresses $H_k(\Omega(X \vee Y))$ in terms of $H_i(\Omega(X \vee Y))$ $i < k$. The number of computations does not increase with k. Of course $H_i(\Omega(X \vee Y))$ with $i \leqq 3(n_1 + n_2)_{+4}$ has to be computed independently, for example by the method of [5].

In §2 we state the result of [5] in a generalized form which will be used here. We also present in this section most of the relevant notation of this paper. Recursion formulas in general are introduced in §3. The recursion formula for the free component of $H(\Omega(X \vee Y))$ is presented in §4. In §5 we derive a recursion formula for $H(\Omega X) \amalg H(\Omega Y)$. The main result which is a recursion formula for the torsion component of $H(\Omega(X \vee Y))$ is proved in §6. We close with an application by computing $H(\mathrm{SO}_3 * \mathrm{SO}_3)$.

The ring R will always be a principal ideal domain. The notation and terminology are those of [5].

2. The holomogy of $\Omega(X \vee Y)$ in dimension k. Let L^j be