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NORMAL STRUCTURE COEFFICIENTS FOR BANACH SPACES

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This paper introduces several geometric concepts in Banach spaces related to normal structure and uses these ideas to generalize a recent theorem of Baillon for fixed points of nonexpansive mappings.

1. **Definitions.** The concepts introduced in this paper are phrased in terms of reflexive Banach spaces. This is not a serious restriction, but rather one of technical convenience. All of the concepts discussed here deal with closed bounded convex subsets of a reflexive Banach space, so one could replace "closed bounded and convex" with "weakly compact and convex", if desired.

For the following definitions, X will denote a reflexive Banach space and C will denote a closed bounded convex subset of X.

For each x in C, let $r(x, C) = \sup\{||x - y||: y \text{ in } C\}$ and let $R(C)$ denote the *Čebyšev radius of* C [11, p. 178]:

$$R(C) = \min\{r(x, C): x \text{ in } C\} .$$

This minimum is achieved because of the weak compactness of C. Let $D(C)$ denote the *diameter of* C, $D(C) = \sup\{||x - y||: x, y \text{ in } C\}$.

A space X has *normal structure* [2] provided that for each closed bounded convex subset C of X with more than one member, $R(C) < D(C)$.

The three Banach space coefficients defined in the following paragraphs are the principal objects of study in this paper.

The *normal structure coefficient of* X, denoted by $N(X)$, is the infimum of the set of number $D(C)/R(C)$ taken over all closed convex subsets C of X with more than one member.

The *asymptotic diameter of a bounded sequence* $\{x_n\}$ in X is defined to be $\lim_n \sup\{||x_m - x_k||: m \geqq n, k \geqq n\}$. The *bounded sequence coefficient of* X, denoted $BS(X)$, is the supremum of the set of all numbers M with the property that for each bounded sequence $\{x_n\}$ with asymptotic diameter A, there is some y in the closed convex hull of the sequence such that $M \cdot \limsup_n ||x_n - y|| \leqq A$.

The *weakly convergent sequence coefficient of* X, denoted by $WCS(X)$, is defined similarly, replacing "bounded" with "weakly convergent".

These three coefficients are related to normal structure in that if any one of the three is greater than 1 (all are between 1 and 2), the space has normal structure; this result is established in § 2. An