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A NOTE ON TAMELY RAMIFIED POLYNOMIALS

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Let $f(x)$ be a monic polynomial with coefficients in a Dedekind ring A. If P is a prime ideal and A_P denotes the completion of A at P then $f(x)$ is said to be integrally closed at P if $A_P[X]/(f(X))$ is isomorphic to a product of discrete valuation rings. The purpose of this note is to show that if $f(x)$ appears to be tamely ramified and integrally closed at P (in terms of its discriminant and factorization mod P) then in fact it is.

If $f(\alpha) = 0$, where $f(x)$ is a monic irreducible polynomial with coefficients in $\boldsymbol{Z}$, then the ring $\boldsymbol{Z}[\alpha]$ is of finite index in the ring R of algebraic integers in $\boldsymbol{Q}(\alpha)$. The full ring of integers can be obtained by applying a very general algorithm due to Zassenhaus ([6]). There are well known cases where this is unnecessary. If, for instance, $f(x)$ is an Eisenstein polynomial at p, or if p^2 does not divide the discriminant of $f(x)$, then the polynomial $f(x)$ is integrally closed at p (which is equivalent to saying that p does not divide the index $[R: \boldsymbol{Z}[\alpha]]$). The theorem below asserts that if the power of p that divides the discriminant of $f(x)$ is consistent with the factorization of $f(x)$ mod P and the hypothesis that R is tamely ramified at p, then $f(x)$ is integrally closed at p.

If P is a prime ideal in the Dedekind ring A let $v_P: A \to \boldsymbol{Z} \cup \{\infty\}$ denote the corresponding normalized valuation. Let $d(g)$ and $\mathrm{Disc}\,(g)$ denote the degree and discriminant of a polynomial $g(x)$.

THEOREM. *Suppose that $f(x) \in A[x]$ is a monic polynomial that satisfies*

(a) $f(x) \equiv \Pi g_i(x)^{e_i} \bmod P$

(b) $v_P\,(\mathrm{Disc}\,(f)) = \Sigma_i\,(e_i - 1)d(g_i)$

where the $g_i(x) \in (A/P)[x]$ are distinct monic, irreducible and separable polynomials. Then $f(x)$ is integrally closed at P. Moreover, $p \nmid e_i$ and $A_P[x]/(f(x))$ is isomorphic to a product of discrete valuation rings that are tamely ramified over A_P.

The proof given in the third section is an easy consequence of a purely local result given in the second section. The first section recalls some basic formulas concerning resultants.

REMARKS. (1) It is a standard fact that if $f(x)$ is integrally closed and tamely ramified at P then conditions (a) and (b) must