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PIECEWISE CATENARIAN AND GOING BETWEEN RINGS

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The main purpose of this paper is to prove the following theorem. Let R be a noetherian ring and n a nonnegative integer. Then $R[X_1, \cdots, X_n]$ is a going-between ring (=GB) iff R is GB and is $(n+1)$-piecewise catenarian.

In [7] Ratliff proved that all polynomial rings over an unitary commutative noetherian going-between-(= GB)-ring R are again GB iff R is catenarian (thus universally catenarian by [6, (3.8)] and [5, (2.6)]). (Recall that R is called a GB-ring if for any integral extension R' of R each adjacent pair of Spec (R') retracts to an adjacent pair of Spec (R).)

In the meantime we showed that there are noetherian GB-rings which are not catenarian, thus giving a negative answer to a corresponding question of [7] (s. [2]). So it may be interesting to know more about the relations between the GB-property of polynomial rings and the chain structure of Spec (R). In this note we shall investigate such a relation, thereby improving Ratliff's above result.

To formulate our statement, let us give the following

DEFINITION 1. R is called *n-piecewise catenarian* ($= C_n$). If $(R/P)_{\mathscr{Q}}$ is catenarian for any pair P, Q of Spec (R) related by a saturated chain $P = P_0 \subsetneqq P_1 \subsetneqq \cdots \subsetneqq P_i = Q$ of length $i \leqq n$.

Our main goal is to prove

THEOREM 2. *Let R be a noetherian ring and n a nonnegative integer. Then $R[X_1, \cdots, X_n]$ is* GB *iff R is* GB *and satisfies the property C_{n+1}.*

Noticing that R is catenarian iff it is C_n for all $n > 1$, this gives immediately the quoted result of Ratliff.

To prove 2, let us introduce the following notations

3. (i) $c(R)$ = set of lengths of maximal chains $P_0 \subsetneqq P_1 \subsetneqq \cdots$ of Spec (R) (s. [3], where $c(R)$ was investigated).

(ii) If R is semilocal with Jacobson radical J, put $\hat{d}(R) = \min\{\dim(\hat{R}/\hat{P})$, where $\hat{P}$ is a minimal prime of $\hat{R}\}$, $\hat{R}$ denoting the J-adic completion of R (s. [1]).