

REPRESENTATIONS NAIMARK-RELATED TO *-REPRESENTATIONS; A CORRECTION

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Let A be a Banach $*$ -algebra. A theorem is proved concerning a sufficient condition for a continuous representation of A on a Hilbert space H to be Naimark-related to a $*$ -representation of A on H . One corollary of this result is that a continuous (topologically) irreducible representation of A on H is Naimark-related to a $*$ -representation of A on H if and only if some coefficient of the representation is a nonzero positive functional of A .

One purpose of the paper is to correct in part a previously published result the proof of which contains a serious gap.

1. Introduction. Professor John Bunce has brought to my attention a gap in the proof of Theorem 3 in my paper [2]. Briefly, the problem is as follows: Let A be a Banach $*$ -algebra, and let π be a continuous essential representation of A (not in general a $*$ -representation) on a Hilbert space H . What is established at the beginning of the proof of [2, Theorem 3] is the existence of a π -invariant subspace H_0 of H and an inner-product on H_0 with the property that

$$\langle \pi(f)\xi, \eta \rangle = \langle \xi, \pi(f^*)\eta \rangle \quad (\xi, \eta \in H, f \in A).$$

At this point in the proof results are applied to this inner-product which may be applied only when this form is closable on H ; see [4, pp. 313-315]. It is not proved in [2] that this form is closable, hence the gap in the proof. Although Theorem 3 and its corollaries have not been established in [2], we know of no counter-examples to these statements. All of the results of [2] outside of § 4 (including Theorem 1 and Theorem 7) are to our knowledge correct.

The aim of this note is to partially correct the error. Here we prove a result similar to [2, Theorem 3], and derive from it several corollaries. In particular, it is shown that a continuous, essential, (topologically) cyclic, separable representation of a C^* -algebra is Naimark-related to a $*$ -representation of the algebra; and that a continuous (topologically) irreducible representation of a Banach $*$ -algebra A is Naimark-related to a $*$ -representation of A if and only if some nonzero coefficient of the representation is a positive functional on A .

2. The results. We use the same notation as in [2]. In parti-