PACIFIC JOURNAL OF MATHEMATICS
Vol. 87, No. 1, 1980

AXIOMS FOR CLOSED LEFT IDEALS IN A C^*-ALGEBRA

JEN-CHUNG CHUAN

A set of axioms is formulated to describe the conditions under which a Banach algebra may be embedded as a closed left ideal in a C^*-algebra.

In this paper we attempt to characterize the class of all closed left ideals in a C^*-algebra as a class of Banach algebras equipped with a certain (nonassociative) multiplication structure. To describe such a multiplication, we formulate a set of axioms which extracts the essential properties of the binary operation

$$(x, y) \longrightarrow y^*x$$

taking place in a closed left ideal of a C^*-algebra. Following the notion of centralizers of C^*-algebras introduced by B. E. Johnson [3] and R. C. Busby [1] we are able to show that the axioms are indeed suitable for our purpose: in order that a Banach algebra L fulfills the conditions of the axiom, it is necessary and sufficient that L can be identified with a closed left ideal of some C^*-algebra. This paper is taken from parts of author's Ph. D. thesis under the supervision of C. Akemann.

AXIOM 1. Let $(L, \|\ \|)$ be a complex Banach algebra which contains a closed subalgebra B that has a C^*-algebra structure, i.e., besides the algebraic and the norm structures inherited from L, B has an involution $*$ so that B is a Banach $*$-algebra satisfying $\|x^*x\| = \|x\|^2$ for $x \in B$. Suppose that

$$[\cdot, \cdot]: L \times L \longrightarrow B$$

is a function such that for elements x, y, z in L and for each complex scalar λ the following rules hold:

(i) $[x, y] = [y, x]^*$
(ii) $[x + y, z] = [x, z] + [y, z]$
(iii) $[\lambda x, y] = \lambda[x, y]$
(iv) $[x, x]$ is a positive element of the C^*-algebra B
(v) $\|[x, x]\| = \|x\|^2$
(vi) $\|[x, y]\| \leqq \|x\| \|y\|$
(vii) $[xy, z] = [y, [z, x]]$
(viii) $[x, y] = y^*x$ for x, y in B.

We now exhibit a situation in which the conditions stated in the