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THE RADON-NIKODYM-PROPERTY, σ-DENTABILITY AND MARTINGALES IN LOCALLY CONVEX SPACES

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In this paper we give relations between the Radon-Nikodym-Property (RNP), in sequentially complete locally convex spaces, mean convergence of martingales, and σ-dentability. (RNP) is equivalent with the property that a certain class of martingales is mean convergent, while σ-dentability is equivalent with the property that the same class of martingales is mean Cauchy. We give an example of a σ-dentable space not having the (RNP). It is also an example of a sequentially incomplete space of integrable functions, the range space being sequentially complete.

1. **Introduction, terminology and notation.** A nonempty subset B of a locally convex space (l.c.s.) (over the reals) is called dentable, if for every neighborhood (nbhd) V of o, there exists a point x in B such that

$$x \notin \overline{\text{con}}\,(B \backslash (x + V))$$

($\overline{\text{con}}$ denotes the closed convex hull). X is called dentable if every bounded subset of X is dentable. When we replace $\overline{\text{con}}$ by σ, where

$$\sigma(A) = \left\{ \sum_{n=1}^{\infty} \lambda_n x_n \,\|\, x_n \in A,\ \forall n \in N,\ \sum_{n=1}^{\infty} \lambda_n = 1,\ \sum_{n=1}^{\infty} \lambda_n x_n \text{ convergent},\ \lambda_n \geqq 0 \right\},$$

we get the corresponding definitions for σ-dentability.

We use the following integral:

Let X be a sequentially complete l.c.s., and (Ω, Σ, μ) a finite complete positive measure space.

A function $f: \Omega \to X$ is said to be μ-integrable, if there exists a sequence $(f_n)_{n=1}^{\infty}$ of simple functions such that:

(i) $\lim_n f_n(\omega) = f(\omega)$, μ − a.e..

(ii) For every continuous seminorm p on X:

$$\lim_n \int_\Omega p(f_n(\omega) - f(\omega))d\mu(\omega) = 0 .$$

Put $\int_A f d\mu = \lim_n \int_A f_n d\mu$, $\forall A \in \Sigma$. This limit exists and is in X. Denote $L_X^1(\mu, \Sigma)$ as the space of classes $[f]$ of μ-integrable functions, where $[f] = [g]$ iff $f = g$, μ − a.e..

Put $q(f) = \int_\Omega p(f)d\mu$, where p is any continuous seminorm on X.