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HOLOMORPHIC MAPPING OF PRODUCTS OF ANNULI IN C^n

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Let $\Omega_1, \Omega_2 \subset C^n$ be bounded pseudoconvex Reinhardt domains with the property that $z_1 \cdots z_n \neq 0$ for all $(z_1, \cdots, z_n) \in \bar{\Omega}_j$. A holomorphic mapping $f: \Omega_1 \to \Omega_2$ is discussed in terms of the induced mapping on homology $f_*: H_1(\Omega_1, \boldsymbol{R}) \to H_1(\Omega_2, \boldsymbol{R})$. Specifically, there is a norm on $H_1(\Omega_j, \boldsymbol{R})$ which must decrease under f_*. As a consequence we prove that a domain Ω as above is rigid in the sense of H. Cartan: if $f: \Omega \to \Omega$ is holomorphic and $f_*: H_1(\Omega, \boldsymbol{R}) \to H_1(\Omega, \boldsymbol{R})$ is nonsingular, then f is an automorphism.

1. Introduction. Let $A(R_j) = \{z \in \boldsymbol{C}: 1/R_j < |z| < R_j\}$ be an annulus in the complex plane. If $f: A(R_1) \to A(R_2)$ is a holomorphic mapping, then the topological behavior of f is restricted in terms of the moduli R_1 and R_2 (see Schiffer [6] and Huber [4]). With the methods of Landau and Osserman [5] it will be possible to generalize this result to certain domains which are (topologically) the products of plane annuli. Domains satisfying (2) are also shown to be rigid; see Theorem 2 and Remark 1. In [1] the homology group H_{2n-1} was used to prove rigidity; here we discuss H_1.

Let $\Omega \subset \boldsymbol{C}^n$ be a complex manifold and let

$$\mathscr{F} = \{u \in C^2(\Omega),\ 0 < u < 1,\ u \text{ pluriharmonic}\}\ .$$

If $\gamma \in H_1(\Omega, \boldsymbol{R})$ is a homology class, then a seminorm on γ may be defined by

$$N\{\gamma\} = \sup_{u \in \mathscr{F}} \int_\gamma d^c u \tag{1}$$

where $d^c = i(\bar{\partial} - \partial)$, (see Chern, Levine, and Nirenberg [2]). If $F: \Omega_1 \to \Omega_2$ is a holomorphic mapping, then the map on homology $F_*: H_1(\Omega_1, \boldsymbol{R}) \to H_1(\Omega_2, \boldsymbol{R})$ must decrease this norm.

2. Computation of the intrinsic norm. We will compute this norm for domains $\Omega \subset \boldsymbol{C}^n$ satisfying

(2) Ω is connected, bounded, pseudoconvex, Reinhardt (i.e., $(e^{i\theta_1}z_1, \cdots, e^{i\theta_n}z_n) \in \Omega$ if $z \in \Omega$ and $\theta_1, \cdots, \theta_n \in \boldsymbol{R}$), and if $z \in \bar{\Omega}$, then $z_1 \cdots z_n \neq 0$.

Let $\omega \subset \boldsymbol{R}^n$ be the logarithmic image of Ω, i.e.,