

## A-HOMOLOGY COBORDISM BUNDLES

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**Let  $K$  be a set of primes and  $A$  the localization of the integers away from  $K$ . In this paper we compute the homotopy types of  $G(K)/H(K)$  and  $H(K)/PL$ , when  $H(K)$  is the classifying monoid for  $A$ -homology cobordism bundles, with applications to the space  $BH(K)$ .**

**1. Introduction.** Let  $A$  be a subring of  $\mathbb{Q}$  with unit, and  $K$  the set of primes invertible in  $A$ . This paper is concerned with  $A$ -homology cobordism bundles, which are defined as in [13], using  $A$ -coefficients throughout.

In §2, we define  $A$ -homology cobordism sphere, disc and cone bundles and discuss their basic properties, including representability, existence of normal bundles and transversality. Most of the results of this section are known in some form (from the bundle theories of [19], [28], straightforward generalizations of the  $A = \mathbb{Z}$  case in [13], [15], [8] or special cases of [9], [22]).

In §3, we consider rational surgery obstructions for simply connected manifolds. Our main result is a product formula for  $\mathbb{Z}/m$ -manifolds, which allows us to apply the Morgan-Sullivan construction [18].

In §4 we compute the homotopy type of  $G(K)/H(K)$ . A similar construction has been briefly sketched by Quinn [19], following, as does the one given here, the construction of Sullivan [24] for  $G/\widetilde{PL}$ . We show that  $G(K)/H(K) \cong A^+ \times K(\widetilde{\psi}_3^K \otimes A, 4) \times Y$ , where  $Y$  is given by the fiber diagram

$$\begin{array}{ccc} Y & \longrightarrow & (\prod_{i>0} K(L_i(1; A) \otimes A, i))_{(2)} \\ \downarrow & & \downarrow \\ (BO_K)_2 & \longrightarrow & \prod_{i>0} K(\mathbb{Q}, 4i) . \end{array}$$

Here  $\psi_n^K$  denotes the group of PL  $A$ -homology  $n$ -spheres, modulo  $H_K$ -cobordism, and  $\widetilde{\psi}_3^K$  is the kernel of an invariant  $\widetilde{\psi}_3^K \rightarrow (\mathbb{Z}/8) \otimes A$ .

In §5, we compute the homotopy type of  $H(K)/\widetilde{PL}$ . Our result is:  $H(K)/\widetilde{PL} \cong (\text{BSPL})^{(K)} \times \prod_{i>0} K(\psi_i^K \otimes A, i)$ , where  $(\text{BSPL})^{(K)}$  is the fiber of  $\text{BSPL} \rightarrow \text{BSPL}^K$ .

Finally, in §6, we consider applications to  $A$ -homology cobordism bundles. The homotopy groups of  $BH(K)$  are shown to be