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AN ANALOGUE OF MOREAU'S PROXIMATION THEOREM, WITH APPLICATION TO THE NONLINEAR COMPLEMENTARITY PROBLEM

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This paper concerns the problem of minimizing a convex function subject to nonnegativity constraints, an associated nonlinear complementarity problem, and a new approach to solving these problems. The approach involves solving a sequence of nicer problems which approximate the given ones better and better, and our focus is on certain natural "trajectories" of solutions of the approximating problems. Existence, characterization, and continuous dependence of the solutions is obtained by establishing a complete analogue of Moreau's Proximation Theorem. From this analogue also follow two new facts about the geometric nature of the graphs of subdifferentials in R^n, as well as new information about monotone conjugacy for coordinatewise nondecreasing convex functions on the nonnegative orthant. The largest part of the paper is then devoted to developing a number of rather strong properties of the solution trajectories, particularly as regards the nature of their convergence. Perhaps the most striking property is that these trajectories will locate a maximal strictly complementary solution which, furthermore, can be arranged to have a certain prescribed strong Pareto optimality property. The arithmetic-geometric mean inequality enters decisively at several key points, and the proofs generally rely strongly upon the techniques of finite-dimensional convex analysis.

1. Introduction.

Consider the optimization problem

$$(P_0) \qquad \min_{x \in Q} \{f(x)\} ,$$

where f is a closed proper convex function on R^n and $Q = \{x \in R^n \,|\, x_k \geqq 0 \ \forall k\}$. The simplest and most generally useful convex dual problem to (P_0) is

$$(P_0^*) \qquad \min_{y \in Q} \{f^*(y)\} ,$$

where f^* is the Fenchel conjugate of f, i.e.,

$$f^*(y) = \sup_x \{\langle x, y\rangle - f(x)\}$$

(here $\langle x, y\rangle$ denotes the usual dot product $\sum x_k y_k$). Associated with