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RESOLUTIONS ON THE LINE

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It has been shown previously that under suitable conditions a bounded open set on the line may be resolved into a countable union of disjoint open intervals. Here, such a resolution is obtained for an unbounded open set; it requires the introduction of a suitable system of extended real numbers. The methods used are those of modern constructive analysis.

1. Introduction. A recurring procedure in analysis is that of measuring the distance from a point to a set. It is *not* always possible to do this constructively. Often enough, however, it is possible. Subsets G of a metric space X, for which the distance

$$\rho(x, G) \equiv \inf \{\rho(x, y): y \in G\}$$

exists for every x in X, are called *located*. Metric spaces commonly used have sufficiently many located sets to allow the constructivization of analysis to be carried out in Bishop's *Foundations of Constructive Analysis* [1] and in the work of many others. The *metric complement* of a located set G is the set

$$-G \equiv \{x \in X: \rho(x, G) > 0\} .$$

These metric complements are called *colocated*; they are the open sets with which we shall work, and may be considered to be those open sets having constructive significance.

Our main result (Theorem 6) is that every colocated set on the line is a countable union of disjoint open intervals. Classically (i.e., nonconstructively) this is true for every open set, but *not* constructively; a counterexample is given in [3, §7]. The resolution of a colocated set given here will be developed further in [4] to yield a complete characterization of located sets on the line and an explicit procedure for their construction.

The italicized word "*not*" used in the above paragraphs has a special meaning in constructive mathematics. It means that a counterexample exists in the sense of Brouwer. Such a counterexample consists of a proof showing that the statement in question implies one of several principles which are constructively unbelievable. That is, no proofs of the principles are known, and it seems very unlikely that constructive proofs will ever be found. For example, the statement "every open set on the line is a countable union of disjoint open intervals" implies the *limited principle of*