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TWO QUESTIONS ON WALLMAN RINGS

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In this paper we give an example of a Wallman ring $\mathscr{A}$ on a topological space X such that the associated compactification $\omega(X, Z(\mathscr{A}))$ is disconnected and $\mathscr{A}$ is not a direct sum of any two proper ideals, herewith solving a question raised by H. L. Bentley and B. J. Taylor. Also, an example of a uniformly closed Wallman ring which is not a sublattice is given.

I. Introduction. Biles [2] has called a subring $\mathscr{A}$ of the ring $C(X)$, of all real-valued continuous functions on a topological space X, a Wallman ring on X whenever $Z(\mathscr{A})$, the zero-sets of functions belonging to $\mathscr{A}$, forms a normal base on X in the sense of Frink.

H. L. Bentley and B. J. Taylor [1] studied relationships between algebraic properties of a Wallman ring $\mathscr{A}$ and topological properties of the compactification $\omega(X, Z(\mathscr{A}))$ of X. They proved that if $\mathscr{A}$ is a Wallman ring on X such that $\mathscr{A} = \mathscr{B} \oplus \mathscr{C}$ where $\mathscr{B}$ and $\mathscr{C}$ are proper ideals of $\mathscr{A}$, then $\omega(X, Z(\mathscr{A}))$ is disconnected. We shall prove that the converse of this result is not valid. But, when $\omega(X, Z(\mathscr{A}))$ is disconnected we find a Wallman ring $\mathscr{A}^\circ$, equivalent to $\mathscr{A}$, which is a direct sum of any two proper ideals.

It is well-known that every closed subring of $C^*(X)$, the ring of all bounded functions in $C(X)$, that contains all the rational constants is a lattice. But this is not true for arbitrary closed subrings of $C(X)$. We give an example of a uniformly closed Wallman ring on a space Y which is not a sublattice of $C(Y)$. This corrects an assertion stated in ([1], p. 27).

II. Definitions and basic results. All topological spaces under consideration will be completely regular and Hausdorff. A nonempty collection $\mathscr{F}$ of subsets of a nonempty set X is said to be a ring of sets if it is closed under the formation of finite unions and finite intersections. The collection $\mathscr{F}$ is said to be disjunctive if for each closed set G in X and point $x \in X \sim G$ there is a set $F \in \mathscr{F}$ satisfying $x \in F$ and $F \cap G = \varnothing$. It is said to be normal if for F_1 and F_2 in $\mathscr{F}$ with empty intersection there exist G_1 and G_2 which are complements of members of $\mathscr{F}$ satisfying $F_1 \subset G_1$, $F_2 \subset G_2$ and $G_1 \cap G_2 = \varnothing$. The collection $\mathscr{F}$ is a normal base for the topological space X in case it is a normal, disjunctive, ring of sets that is a base for the closed sets of X.

Throughout this section $\mathscr{D}$ will denote a disjunctive ring of closed