PACIFIC JOURNAL OF MATHEMATICS
Vol. 88, No. 1, 1980

EXTENSIONALITY AND CHOICE IN CONSTRUCTIVE MATHEMATICS

MICHAEL BEESON

We study the relationships between two kinds of formal systems which have been proposed for formalizing modern constructive mathematics. Essentially, we show how to interpret the set-theoretic systems of Friedman and Myhill in the operation-theoretic systems of Feferman. As a by-product of this interpretation, we prove that Friedman's system $\underline{B}$ and certain of Feferman's systems are conservative over intuitionistic arithmetic. We also hope the interpretation casts some light on the nature of the concepts axiomatized in the two systems.

We now explain more leisurely the background of the results and methods of this paper. Ten years ago, Bishop published a book showing that vast portions of modern analysis can be systematically constructivized. His methods involved only careful definitions of the mathematical concepts involved, and a restriction that no proof by contradiction be allowed. That is, he departed from the older school of intuitionism, which introduced such nonclassical ideas as choice sequences, and along with them, some axioms which are not classically valid. Because of this, each of Bishop's theorems is classically valid; this has made the "new constructivism" considerably more appealing to mathematicians than its predecessors. It was, however, a challenge to logicians to find suitable formal systems in which the work of the new constructivists could be carried out. Though Bishop's book is entitled *Foundations of Constructive Analysis*, the title is appropriate only in one sense of the word "foundations"; a thorough philosophical analysis of the conceptions underlying constructivity is still lacking. The same, of course, is more or less true of classical set theory. Several logicians set out to do for the new constructivism what Zermelo, Frankel, and Russell did for classical mathematics; namely, to give formal systems encompassing all the usual arguments, and based on some rough intuition of the underlying ideas. These formal systems should in turn serve to sharpen our understanding of the foundations of the subject.

There has not yet emerged a single formal system for the new constructivism which is as universally recognized as adequate for its purpose as is ZF for classical set theory. There are still several different approaches being discussed. Two approaches which have received the most attention are those of Feferman and Friedman. In order to say more, and especially in order to explain the title of