PACIFIC JOURNAL OF MATHEMATICS
Vol. 88, No. 2, 1980

# ON THE BEHAVIOR OF A CAPILLARY SURFACE AT A RE-ENTRANT CORNER

NICHOLAS J. KOREVAAR

**Changes in a domain's geometry can force striking changes in the capillary surface lying above it. Concus and Finn [1] first studied capillary surfaces above domains with corners, in the presence of gravity. Above a corner with interior angle $\theta$ satisfying $\theta < \pi - 2\gamma$, they showed that a capillary surface making contact angle $\gamma$ with the bounding wall must approach infinity as the vertex is approached. In contrast, they showed that for $\theta \geqq \pi - 2\gamma$ the solution $u(x, y)$ is bounded, uniformly in $\theta$ as the corner is closed. Since their paper appeared, the continuity of $u$ at the vertex has been an open problem in the bounded case. In this note we show by example that for any $\theta > \pi$ and any $\gamma \neq \pi/2$ there are domains for which $u$ does not extend continuously to the vertex. This is in contrast to the case $\pi > \theta > \pi - 2\gamma$; here independent results of Simon [5] show that $u$ actually must extend to be $C^1$ at the vertex.**

We consider bounded domains $\Omega$ in $\boldsymbol{R}^2$ with piecewise smooth boundaries $\partial\Omega$, and functions $u(x, y)$ satisfying

( i ) $\operatorname{div} Tu = 2H(u) = \kappa u$ in $\Omega$; $Tu = Du/\sqrt{1 + Du^2}$, $H(u) =$ mean curvature of the surface $z = u(x, y)$, $\kappa > 0$.

( ii ) $Tu \cdot n = \cos\gamma$ on the smooth part of $\partial\Omega$; $0 \leqq \gamma \leqq \pi$, $n =$ exterior normal to $\partial\Omega$.

Physically $u$ describes the capillary surface formed when a vertical cylinder with horizontal cross section $\Omega$ is placed in an infinite reservoir of liquid having rest height $z = 0$. Then

$$\kappa = \frac{\rho g}{\sigma},$$

where

$\rho =$ density of liquid
$g =$ (downward) acceleration of gravity
$\sigma =$ surface tension between liquid and air.

$$\cos\gamma = \frac{\sigma_1}{\sigma},$$

where

$\sigma_1 =$ surface attraction between liquid and cylinder.

Geometrically $\gamma$ is the contact angle between the capillary surface and the bounding cylinder; it is the angle between the downward