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GENERALIZED SOLUTIONS FOR THE MEAN CURVATURE EQUATION

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The purpose of this paper is to discuss general boundary valve problems for the mean curvature equation

$$\text{div}\, Tu = H(x, u)\ ; \quad Tu = Du/\sqrt{1+|Du|^2} \tag{0.1}$$

in a bounded domain $\Omega \subset \boldsymbol{R}^n$. More precisely, we shall consider the problem of minimizing the functional

$$\mathscr{F}(u) = \int_\Omega \sqrt{1+|Du|^2} + \int_\Omega \lambda(x, u)dx + \int_{\partial\Omega} \kappa(x, u)dH_{n-1} \tag{0.2}$$

where

$$\lambda(x, u) = \int_0^u H(x, t)dt\ . \tag{0.3}$$

It is easily seen that (0.1) is the Euler equation of the functional $\mathscr{F}$. The third integral in (0.2) describes the boundary conditions: if $u \in C^1(\bar{\Omega})$ and κ is of class C^1 we have

$$Tu \cdot \nu = \gamma(x, u) \qquad \text{on} \quad \partial\Omega \tag{0.4}$$

where ν denotes the interior normal to $\partial\Omega$, and

$$\kappa(x, u) = \int_0^u \gamma(x, t)dt\ . \tag{0.5}$$

When κ is not differentiable, as it is the case for the Dirichlet problem, condition 0.4 does not hold any longer, and we have instead the weaker condition

$$\gamma^-(x, u) \leqq Tu \cdot \nu \leqq \gamma^+(x, u) \tag{0.6}$$

on $\partial\Omega$, where

$$\gamma^\pm(x, u) = \lim_{t \to u^\pm} \gamma(x, t)\ .$$

For example, for the Dirichlet problem with boundary datum $f(x)$ we have

$$\kappa(x, u) = |u - f(x)| - |f(x)|$$

and

$$\gamma(x, t) = 1 - 2\varphi_F(x, t)$$

where φ_F is the characteristic function of the subgraph F of f: