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ON EQUISINGULAR FAMILIES OF ISOLATED SINGULARITIES

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Basic properties of a definition of equisingularity for families of (algebraic, analytic or algebroid) varieties, singular along a given section, are studied. The equisingularity condition is: given a family $p: X \to Y$, with a section s, it is required that the natural morphism $E \to Y$ be flat, where E is the exceptional divisor of the blowing-up of X with center the product of the ideal defining s and the relative Jacobian ideal.

The following results hold: (a) This condition is invariant under base change (b) It implies equimultiplicity, the validity of the Whitney conditions and topological triviality along s (c) If Y is reduced, the condition holds over a dense open set of Y, whose complement is a subvariety of Y (d) If Y is smooth and the fibers of p are plane curves, this definition agrees with Zariski's.

Introduction. In this article, we discuss some basic consequences of a possible definition of equisingularity, for families of isolated singularies of algebraic, algebroid and analytic varieties (cf. Definitions (1.1) and (1.4)). Our basic definition is closely related to one suggested by Hironaka years ago. In fact, around 1964, in his pioneering work on the Whitney conditions, he did the following: given a family of isolated singularies $\pi: X \to Y$ (say, to simplify, with Y smooth), he took the blowing-up Z of X with center IJ, where the ideal I defines the singular locus of X, and J is the Jacobian ideal of π, and after that the normalization $\widetilde{Z}$ of Z. If $\widetilde{E}$ is the subspace of $\widetilde{Z}$ corresponding to $IJ\mathscr{O}_{\widetilde{Z}}$, he required that the composed morphism $\widetilde{h}: \widetilde{E} \to Y$ be flat (cf. [5]). Since then, some interesting topological results were obtained with these techniques (cf. [9]); however, apparently no careful study of a theory of equisingularity based on these ideas has been attempted. Here we study some basic results related to the definition that is obtained when, in Hironaka's process, the normalization is omitted. We call this "condition $\mathscr{E}$". This seems a reasonable requirement, if we are interested in having good functorial properties and in accepting nonreduced spaces (e.g., infinitesimal deformations). More precisely, in §1 we present the basic definitions, and we prove that this theory has some of the fundamental "nice" properties that a good theory of equisingularity should have (equimultiplicity, topological triviality, openness, etc). The proofs of §1 are simple applications of known