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LOCAL Λ SETS FOR PROFINITE GROUPS

M. F. HUTCHINSON

Let E be a subset of the dual $\hat{G}$ of a profinite group G. It is shown that if E is a local Λ set then the degrees of the elements of E must be bounded. It follows that $\hat{G}$ contains an infinite Sidon set if and only if $\hat{G}$ has infinitely many elements of the same degree. This characterisation is the same as one previously obtained for compact Lie groups.

Preliminaries. Let G be a compact group with normalized Haar measure λ_G. For $p \in [1, \infty[$ the Banach space of pth power integrable complex-valued functions on G is denoted $(L^p(G), \|\cdot\|_p)$. The *dual object* $\hat{G}$ of G is taken to be a maximal set of pairwise inequivalent continuous irreducible unitary representations of G. For each $\sigma \in \hat{G}$ let d_σ be the *degree* or dimension of the representation space of σ and let χ_σ denote its *trace*. The *Fourier transform* of $f \in L^1(G)$ is the matrix-valued function $\hat{f}$ on $\hat{G}$ defined by

$$\hat{f}(\sigma) = \int_G f(x)\sigma(x^{-1})d\lambda_G(x) \qquad (\sigma \in \hat{G}) .$$

If E is a subset of $\hat{G}$ let $S_E(G)$ denote the set of all trigonometric polynomials on G whose Fourier transforms are supported by just one element of E. For $p \in]1, \infty[$ call E a *local Λ_p set* if there exists a positive constant κ such that

$$\|f\|_p \leqq \kappa\|f\|_1$$

for all $f \in S_E(G)$. Call E a *local central Λ_p set* if there exists a positive constant κ such that

$$\|\chi_\sigma\|_p \leqq \kappa\|\chi_\sigma\|_1$$

for all $\sigma \in E$. Further, E is a *local Λ set* if there exists a positive constant κ such that

$$\|f\|_p \leqq \kappa p^{1/2}\|f\|_2$$

for all $f \in S_E(G)$ and all $p \in]2, \infty[$. A local Λ set is local Λ_p for every $p \in]1, \infty[$. See §37 of [4] for a general introduction to the theory of lacunary sets.

If G is *profinite* and $\{N_\alpha\}_{\alpha \in A}$ is a neighborhood base at the identity consisting of open normal subgroups of G then each $\sigma \in \hat{G}$ has kernel containing some N_α by Lemma (28.17) of [4]. Thus we