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APPLICATIONS OF TOPOLOGICAL TRANSVERSALITY TO DIFFERENTIAL EQUATIONS I. (SOME NONLINEAR DIFFUSION PROBLEMS.)

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In this paper, topological techniques are used to establish existence results for some boundary value problems arising in diffusion theory. Questions of uniqueness are also treated. Our topological arguments are based on the topological transversality theorem rather than the Leray-Schauder theory. An important feature of our approach is that some of the results obtained cannot be deduced by a direct application of the latter theory. Further applications of topological transversality to differential equations will be given in forthcoming parts of the paper.

0. **Introduction.** In this paper, we treat questions of existence and uniqueness for the solutions to certain systems of differential equations each of which models a steady state, one dimensional diffusion process. Conservation of mass considerations lead to the following system of differential equations for the unknow concentration $C = C(x)$ of the diffusing substance and the velocity $v=v(x)$ of the diffusing medium (see [1]):

$$(\mathscr{P})\qquad \begin{cases} -(D(x, C(x))C'(x))' + (v(x)C(x))' = f(x, C(x), C'(x)),\ 0 \leqq x \leqq 1\ , \\ -D(0, C(0))C'(0) + v_0 C(0) = L,\ C(1) = c_1\ , \\ \qquad v'(x) = -J(x, C(x))\ , \\ \qquad v(0) = v_0\ . \end{cases}$$

Here $D(x, C)$ is the diffusion coefficient which we suppose to be continuously differentiable and to satisfy,

$$(1)\qquad 0 < \varepsilon \leqq D(x, C) \text{ on } [0, 1] \times [0, \infty)\ .$$

Also, c_1, v_0, L are given constants with $c_1, L \geqq 0$, and the source term $f(x, C, C')$ is continuous and satisfies,

$$(2)\qquad \begin{cases} 0 \leqq f(x, C, C') \leqq A + B(|C|^\alpha + |C'|^\alpha)\ , \\ \text{on } [0, 1] \times [0, \infty) \times (-\infty, \infty)\ , \end{cases}$$

where $A, B \geqq 0,\ 0 \leqq \alpha < 1$. Also,

$$||h|| = \max_{0 \leqq x \leqq 1} |h(x)|$$