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AN ANALOGUE OF THE WIENER-TAUBERIAN THEOREM FOR SPHERICAL TRANSFORMS ON SEMI-SIMPLE LIE GROUPS

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Let G be a semi-simple connected noncompact Lie group with finite center and K a fixed maximal compact subgroup of G. Fix a Haar measure dx on G and let $I_1(G)$ denote those functions in $L^1(G, dx)$ which are biinvariant under K. The purpose of this paper is to prove that if $f \in I_1(G)$ is such that its spherical Fourier transform (i.e., Gelfand transform) $\hat{f}$ is nowhere vanishing on the maximal ideal space of $I_1(G)$ and $\hat{f}$ "does not vanish too fast at ∞", then the ideal generated by f is dense in $I_1(G)$. This generalizes earlier results of Ehrenpreis-Mautner for $G=\mathrm{SL}(2, \boldsymbol{R})$ and R. Krier for G of real rank one.

1. **Introduction.** Let f be an L^1-function on $\boldsymbol{R}$ (or more generally on a locally compact abelian group). Then the celebrated Wiener-Tauberian theorem says that if the Fourier transform $\hat{f}$ is a nowhere vanishing function then the ideal generated by f is dense in $L^1(\boldsymbol{R})$. In [1] Ehrenpreis and Mautner observe that the corresponding result is not true if one considers the commutative Banach algebra of K-biinvariant functions on noncompact semi-simple Lie group G, where K is a maximal compact subgroup of G. More precisely, let $G = \mathrm{SL}(2, \boldsymbol{R})$ i.e., the group of 2×2 real matrices of determinant 1, and

$$K = \mathrm{SO}(2) = \left\{ \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} ; \quad 0 \leqq \theta \leqq 2\pi \right\} \text{ and let}$$

$I_1(G)$ denote the commutative Banach algebra of K-biinvariant L^1-functions on G. For $f \in I_1(G)$, let $\hat{f}$ denote its spherical Fourier transform (see § 2). Then Ehrenpreis and Mautner observed that there exist functions $f \in I_1(G)$ such that $\hat{f}$ does not vanish anywhere on the maximal ideal space of $I_1(G)$ and yet the algebra generated by f is *not* dense in $I_1(G)$. However they were able to show that if $\hat{f}$ is non vanishing *and* $\hat{f}$ 'does not go to zero too fast at ∞' then the ideal generated by f is indeed dense in $I_1(G)$. (Theorems 6 and 7 of [1].) These results have been generalized by R. Krier [6] in his thesis when G is a noncompact connected semi-simple Lie group of real rank 1. (The author does not know whether Krier's results have been published.) The purpose of this note is to prove a theorem in the spirit of Theorem 7 of [1] without any restriction