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SOLUTION OF THE MIDDLE COEFFICIENT PROBLEM FOR CERTAIN CLASSES OF C-POLYNOMIALS

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A well-known conjecture states that for polynomials having all their zeros on the unit circle C half the maximum modulus on C bounds the modulus of all the coefficients. This has been established in all cases except for the middle coefficient of even degree polynomials greater than four. In this note this conjecture is verified for all even degree polynomials having simple zeros in a set of arcs dividing the circle into equal parts and related classes of polynomials. The local extremal polynomials are identified.

1. Introduction. Throughout this note polynomials whose zeros all lie on the unit circumference $C = \{z \mid |z| = 1\}$ will be considered and refered to as C-polynomials. If P is a polynomial $M(P) = \text{Max}_{z \in C} |P(z)|$. Also $P^*(z) = z^n \overline{P(1/\bar{z})}$, where n is the degree of P.

The following conjecture due to P. Erdös was stated in [3], and in corrected form in [4].

Conjecture 1. Let

$$P(z) = a_n z^n + \cdots + a_1 z + a_0 \tag{1}$$

be a C-polynomial. Then $|a_i| \leqq M(P)/2$ for $i = 0, 1, \cdots, n$. This conjecture was established in [5] and [6] for all cases except $n = 2k$ and $i = k$. In [6] another conjecture was raised in this connection.

Conjecture 2. If the zeros of $P(z)$ in (1) all lie on the exterior of C then $|a_i| \leqq M(P)/2$ for $n/2 \leqq i \leqq n$. For comparison we add

Conjecture 3. If the degree of $P(z)$ in (1) is even $n = 2k$ then $|a_k| \leqq M(P)/2$.

The special significance of Conjecture 3 is that it actually is equivalent to Conjectures 1 and 2 but its statement is the most economical. This is summarized in

LEMMA 1. *Conjecture* 3 *implies Conjectures* 1 *and* 2.

Proof. (a) If $P(z)$ is a C-polynomial given by (1) then $P^2(z) = c_{2n}z^n + \cdots + c_n z^n + \cdots + c_0$ is an even degree C-polynomial and $c_n =$