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# AN INTRINSIC CHARACTERIZATION FOR PI FLOWS

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**An intrinsic characterization for a minimal flow $(X, T)$ to be a PI flow is given. This characterization is then combined with some recent techniques of R. Ellis to prove the general PI and HPI versions of the Veech Structure Theorem.**

0. Introduction. A pointed minimal flow $(X, x_0, T)$ is PI if there is an ordinal $\Lambda$, a collection of pointed minimal flows $\{(X_\lambda, x_\lambda): \lambda \leqq \Lambda\}$, and a homomorphism $\pi$, $\pi: (X_\Lambda, x_\Lambda) \to (X, x_0)$ such that

(i) $X_0$ is the trivial flow

(ii) for each $\lambda < \Lambda$ there is a homomorphism $\varphi_\lambda^{\lambda+1}$, $\varphi_\lambda^{\lambda+1}: (X_{\lambda+1}, x_{\lambda+1}) \to (X_\lambda, x_\lambda)$ which is either proximal or almost periodic,

(iii) for each limit ordinal $\lambda_0 \leqq \Lambda$, $(X_{\lambda_0}, x_{\lambda_0})$ is inv lim $\{(X_\lambda, x_\lambda): \lambda < \lambda_0\}$, and

(iv) $\pi$ is proximal.

The collection of flows $\{(X_\lambda, x_\lambda): \lambda \leqq \Lambda\}$ and the associated maps $\{\varphi_\lambda^{\lambda+1}\}_{\lambda<\Lambda}$ are called a PI tower for $(X, x_0, T)$. $(X, x_0, T)$ is *strictly* PI if $\pi$ is the identity map. For a discussion of the role of PI flows in topological dynamics, see part 2 of Veech's article [9].

With the exception of the definition, the only condition equivalent to PI in the literature is that the group for the flow, $G(X, x_0)$, contain the group $G_\infty$. (See [10] for a characterization of PD flows.) In this paper we give an intrinsic characterization of PI. Section 1 is devoted to this characterization.

In a recent paper, [3], Ellis proved that the Furstenberg structure theorem holds for any distal flow. Modifying Ellis's technique and applying our characterization we show, in § 2, that for a large class of properties of flows, all flows with one of these properties are PI iff all metric flows with the same property are PI. As a corollary, we show that every point-distal flow is PI. Using this fact we establish that every point-distal flow is actually HPI (the proximal maps in the PI tower are highly proximal) thus proving the General Veech Structure Theorem.

We assume throughout this paper that the reader is familiar with the general theory of PI flows as contained in [5] or [6]. Our notation is primarily that of Glasner's book, [6], with the obvious modification that our group actions are written on the right. In particular, for a fixed topological group $T$, $M(T)$, or just $M$, is a fixed minimal right ideal in $\beta T$ with the usual semi-group structure. $J(T)$, or just $J$, is the set of idempotents in $M(T)$. If $U \subseteq T$ then