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RESTRICTIONS OF PRINCIPAL SERIES TO A REAL FORM

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In this paper I consider restrictions of nondegenerate principal-series representations of a complex semisimple Lie group to a noncompact real form.

The motivation for studying these restrictions comes from several different directions. First it is a problem of some interest to physicists. For example, a low-dimensional instance of my work occurs in [2]. Second the problem and methods of solution are closely related to the project [7], [8]. In fact I shall settle the main outstanding conjecture of [7] in this paper. Other reasons for treating this problem are: restrictions provide a means of realizing discrete-series representations of real semisimple Lie groups (see [3])—in this work we seek to enumerate which discrete series actually occur in which restrictions; it's already well-known (see e.g., the work of Moore [11]) that restrictions can be studied in order to obtain information on ergodicity of group actions on homogeneous spaces. Finally, there is an obvious connection between this problem and the problem of decomposing tensor products of principal-series representations of real semisimple Lie groups (compare Theorem 2.1 here and [9, Theorem 1]). I have some hope that techniques employed in this paper might eventually prove useful in studying tensor products of discrete-series representations.

Here is a brief description of the main results and of the organization of the paper. Let $\mathfrak{G}$ be a complex semisimple Lie group, $G \subseteq \mathfrak{G}$ a real semisimple Lie group whose complexification is $\mathfrak{G}$. The nondegenerate principal series representations of $\mathfrak{G}$ are induced from unitary characters of a Borel subgroup. So let $\mathfrak{B} \subseteq \mathfrak{G}$ be a Borel subgroup, $\chi \in \hat{\mathfrak{B}}$ a unitary character, and form $\pi(\chi) = \mathrm{Ind}_{\mathfrak{B}}^{\mathfrak{G}} \chi$. We are interested in a description of the irreducible components, and their multiplicities, of the representation $\pi(\chi)|_G$. Since $\pi(\chi)$ is an induced representation, we begin our study by invoking the Subgroup Theorem. This requires an explicit knowledge of the $\mathfrak{B}$: G double cosets in $\mathfrak{G}$. But that data has been worked out already in [13]. We shall combine Wolf's results with the Subgroup Theorem to reduce our problem to the study of representations of the form $\mathrm{Ind}_H^G \chi_1$, where χ_1 is a unitary character of a *maximally compact* Cartan subgroup H in G (see Theorem 2.1). This is the content of § 2. We then analyze the spectrum of $\mathrm{Ind}_H^G \chi_1$ by the technique of Anh reciprocity. That requires that we compute the direct integral decomposition of the