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MOIRÉ PHENOMENA IN ALGEBRAIC GEOMETRY: POLYNOMIAL ALTERNATIONS IN R^n

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This paper introduces an object in algebraic geometry akin to but different from an algebraic variety. The main idea is this: The concept of "inverse image under a polynomial map of a point" (=algebraic variety) is replaced by "inverse image under a polynomial map of a periodic subset." In this paper R is the groundfield, and the periodic subset is taken to be $[0,1)+2Z\subseteq R$. These inverse images, which we call *polynomial alternations*, are, in R^2, like diffraction gratings encountered in optics. They are closed under complementation as well as "mod two sum." This sum is like intersection for ordinary varieties in at least one important way — an analogue of the usual dimension theorem holds under mod two sum. Union and intersection are dual, and each gives rise to a phenomenon not encountered with ordinary varieties — namely striations, or "moiré fringes" are formed. These fringes run along algebraic varieties, and these varieties correspond to linear combinations of the polynomials defining the alternations. A density is induced in each algebraic variety, and this natural density is itself periodic. It depends on the coefficients of the linear combination; the author determines this function.

1. Introduction. Suppose that on a transparent sheet one inks in a parallel family of straight bands, to create a "diffraction ruling." Assume that all bands (the inked-in ones as well as the clear ones) have the same width. If another such sheet is placed on the first one, and if the two families are almost (but not exactly) parallel, then there is created a third family of "bands" or "fringes." These fringes will be wider, and more widely-spaced, than the original bands, and almost perpendicular to them. (See Figure 1.) More generally, if the second family of bands differs from the first by a linear transformation T which is close to but not equal to the identity map, then in their union we will see a new set of wider and more widely-spaced fringes, whose orientation depends on T. This phenomenon is not restricted to families of straight bands: Figure 2 shows the moiré phenomenon arising from the union of two slightly-displaced Fresnel zone plates, and Figure 3, from the union of a straight-line plate with a Fresnel plate.

Moiré phenomena have been noticed in the physical sciences, and have found a number of important applications, particularly in