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HOMOTOPY CLASSIFICATION OF LENS SPACES FOR ONE-RELATOR GROUPS WITH TORSION

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Let Ξ be a one-relator group with presentation $\mathscr{R} = (x_1, \cdots, x_n: R^p)$ where R is not a proper power and $p \geqq 2$. Then given any integer q, relatively prime to p, we can construct the Lens space $\mathscr{L}(p, q)$ for Ξ from the cellular model $C(\mathscr{R})$ of the presentation $\mathscr{R}$ by attaching a 3-cell via the attaching map $R^q - 1$, which generates the ideal $Z\Xi(R-1) \approx \pi_2(C(\mathscr{R}))$. In this paper we classify these Lens spaces up to homotopy type. We also discuss the non-cancellation aspect of these Lens spaces.

Introduction. In this paper we are interested in Lens spaces for one-relator groups with torsion. Given relatively prime integers p and q, with $p \geqq 2$, we have the ordinary Lens space $L(p, q)$ with fundamental group finite cyclic of order p. The 2-skeleton of $L(p, q)$ is the cellular model $C(\mathscr{R})$ of the presentation

$$\mathscr{R} = (x: x^p)$$

and $L(q, p)$ is obtained from its 2-skeleton by attaching a 3-cell via the attaching map $x^q - 1$, which generates the ideal $ZZ_p(x-1) \approx \pi_2(C(\mathscr{R}))$. The cellular chain complex of the universal covering $\tilde{L}(p, q)$ of $L(p, q)$ is given by

$$C_*(L(p, q)): ZZ_p \xrightarrow{x^q - 1} ZZ_p \xrightarrow{1 + x + \cdots + x^{p-1}} ZZ_p \xrightarrow{x - 1} ZZ_p$$

where x is a generator of the cyclic group Z_p. J. H. C. Whitehead [11] has shown that $L(p, q)$ and $L(p, r)$ have the same homotopy type if and only if qr or $-qr$ is a quadratic residue mod p. We consider the following analogue: Let Ξ be a one-relator group with presentation

$$(1) \qquad \mathscr{R} = (x_1, \cdots, x_n: R^p)$$

where R is not a proper power and $p \geqq 2$. Then given any integer q, relatively prime to p, we can construct the Lens space $\mathscr{L}(p, q)$ obtained from the cellular model $C(\mathscr{R})$ of the presentation R by attaching a 3-cell via the attaching map $R^q - 1$, which generates the ideal $Z\Xi(R-1) \approx \pi_2(C(\mathscr{R}))$. Clearly the fundamental group of $\mathscr{L}(p, q)$ is isomorphic to Ξ. The cellular chain complex for the universal covering $\tilde{\mathscr{L}}(p, q)$ of the Lens space $\mathscr{L}(p, q)$ is given by

$$C_*(\tilde{\mathscr{L}}(p, q)): Z\Xi \xrightarrow{R^q - 1} Z\Xi \xrightarrow{\partial_2} (Z\Xi)^n \xrightarrow{\partial_1} Z\Xi$$