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WIRTINGER APPROXIMATIONS AND THE KNOT GROUPS OF F^n IN S^{n+2}

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We consider the problem of deciding whether or not a given group G has a Wirtinger presentation, i.e., a presentation in which each defining relation states that two generators are conjugate or that a generator commutes with some word. This property is important because it characterizes those groups that can be realized as knot groups of closed, orientable n-manifolds in S^{n+2}. We isolate the obstruction in the form of an abelian group somewhat related to $H_2(G)$. We do this by considering Wirtinger-presented groups that are approximations to G and prove the existence of a best-approximation.

A group G can be realized as a knot group $\pi_1(S^{n+2} - F^n)$ ($n \geqq 2$), where F^n is a closed, orientable, connected n-manifold tamely embedded in the sphere S^{n+2}, if and only if G satisfies the following:

(1) G is finitely presented.

(2) $G/G' \cong \boldsymbol{Z}$.

(3) There exists $t \in G$ such that $G/\langle\langle t \rangle\rangle = \{1\}$.

(4) G has a Wirtinger presentation (see Definition 0.1).

The necessity of the algebraic conditions may be seen as follows: (1)-(3) are well-known (see e.g., [8] or [9]). (The methods of this paper can be used to develop a theory of Wirtinger approximations for G/G' free abelian of rank m, i.e., F^n having m components, but we restrict ourselves to $m = 1$ to minimize notation and keep the proofs clear.) (4) is well-known for 1-manifolds (not necessarily connected) in S^3 and we proceed by induction on dimension, using the method of slices [4, §6] to present $\pi_1(S^{n+2} - F^n)$. The sufficiency of the algebraic conditions is established by using methods of Yajima [14] (rediscovered by D. Johnson; see [7] for nice exposition) to construct a surface F^2 in S^4 having a given group.

In this paper, we suppose we are given a group G satisfying (1)-(3) and try to decide whether or not G satisfies (4). If we replace (4) by the property $H_2(G) = 0$, we obtain Kervaire's list [8] [9] characterizing the knot groups of spheres $S^n \subset S^{n+2}$. Thus (1)-(3) plus $H_2(G) = 0$ imply (4); a purely algebraic proof of this fact is given in [15], and we shall recover this theorem as Corollary 1.8.

There was some speculation [10, Problem 4.29], [13, Conj. 4.13] that $H_2(G) = 0$ actually is necessary for G to be $\pi_1(S^{n+2} - F^n)$, but counterexamples have been found ([2], [11], Example 3.4 below). When we know $H_2(G) = 0$, G has a Wirtinger presentation in terms