PACIFIC JOURNAL OF MATHEMATICS
Vol. 90, No. 1, 1980

COMPARISON AND OSCILLATION CRITERIA FOR SELFADJOINT VECTOR-MATRIX DIFFERENTIAL EQUATIONS

ROGER T. LEWIS AND LYNNE C. WRIGHT

Let

$$L(y) = \sum_{k=0}^{n} (-1)^k (P_k(x) y^{(k)}(x))^{(k)}$$

where the coefficients are real, continuous, $m \times m$, symmetric matrices, $y(x)$ is an m-dimensional vector-valued function, and $P_n(x)$ is positive definite for all $x \in I$. We consider both the case for which the singularity is at ∞, $I = [1, \infty)$, and the case for which the singularity is at 0, $I = (0, 1]$.

The main theorem is a comparison result in which the equation $L(y) = 0$ is compared with an associated scalar equation. Then, general theorems for the oscillation and nonoscillation of $L(y) = 0$ are presented which can be used when the comparison theorem does not apply. Some of the proofs indicate how scalar oscillation and nonoscillation criteria can be extended to the vector-matrix case when the associated scalar theorem has been proved using the quadratic functional criteria for oscillation. In general, proofs using the associated Riccati equation and other familiar methods do not extend as easily.

1. Introduction. Most of the theorems contained herein appear in the recent Ph. D. dissertation of Wright [25].

For general treatments of the oscillation of $L(y) = 0$, the reader is referred to the lecture notes of Coppel [3] and Kreith [14], the book of Reid [20], the paper of Etgen and Lewis [5], and the references contained therein.

Matrix notation will be used throughout this paper. For example, if A is a matrix with elements a_{ij}, $\int A$ will be the matrix with elements $\int a_{ij}$. Differentiation is defined similarly. $A \geqq B$ is valid if, and only if, $A - B$ is positive semidefinite. The letter I will denote the identity matrix. By $\|A\|$, we shall mean the operator norm of the matrix A which is induced by the Euclidean vector norm, i.e., $\|A\| = \sup \|A\xi\|$ where the supremum is taken over all vectors ξ of norm 1. The notation A^* shall denote the conjugate-transpose of the matrix A and for vectors ξ_1 and ξ_2, $(\xi_1, \xi_2) = (\xi_2^* \cdot \xi_1)^{1/2}$ is the inner product.

If there exists a number $b > a$ such that $L(y) = 0$ has a nontrivial solution satisfying