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# MOIRÉ PHENOMENA IN ALGEBRAIC GEOMETRY: RATIONAL ALTERNATIONS IN $R^2$

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**This paper investigates rational alternations, principally in $R^2$. Rational alternations in $R^n$ generalize the polynomial alternations studied in the author's Moiré Phenomena in Algebraic Geometry: Polynomial Alternations in $R^n$. Rational alternations, like polynomial alternations, have the spirit of diffraction gratings, but may possess singularities, where grating bands flow together. Both alternations carry with them more information than ordinary varieties. As in the polynomial case, the systems of varieties making up two rational alternations generate new systems of varieties under union (or dually, intersection), corresponding to systems of moiré fringes of various orders. This paper investigates density functions naturally associated with these fringes, and studies the behavior of the fringes at points of indeterminacy of the defining functions.**

## 1. Introduction.

For the sake of motivation, we begin not with a general rational function, but with a polynomial. Thus, let $p$ be an element of a polynomial ring $k[X_1, \cdots, X_n]$ over a field $k$. Then $p$ defines an algebraic variety $V(p)=\{(x_1, \cdots, x_n) \mid p(x_1, \cdots, x_n)=0\}$ in $k^n = k_{X_1,\ldots,X_n}$. It is natural to ask how operations on polynomials translate into operations on the associated varieties. For instance, if $p, q \in k[X_1, \cdots, X_n]$, then

$$V(pq) = V(p) \cup V(q) . \tag{1.1}$$

This is especially satisfying because the variety $V(pq)$ is so easily obtained from the original pieces $V(p)$ and $V(q)$, just by taking their union. The simple from in (1.1) of course holds for ideals—that is, for ideals $\mathfrak{a}, \mathfrak{b}$ in $k[X_1, \cdots, X_n]$, we have

$$V(\mathfrak{a}\mathfrak{b}) = V(\mathfrak{a}) \cup V(\mathfrak{b}) . \tag{1.2}$$

What about sum? That is, how is $V(p+q)$ related to $V(p)$ and $V(q)$? $V(p+q)$ does not have that same kind of simple geometric relationship to $V(p)$ and $V(q)$. Of course, one cannot expect a purely geometric answer to this since, for example, $V(p) = V(ap)$ $(a \in k\backslash\{0\})$, but in general $V(p+q) \neq V(ap+q)$. Although "$V(\mathfrak{a}+\mathfrak{b}) = V(\mathfrak{a}) \cap V(\mathfrak{b})$" answers an analogous ideal-theoretic question, there is still a natural polynomial question, and polynomials should not be neglected. The trouble is that since $V(p) = V(ap)$ $(a \in k\backslash\{0\})$, it seems that in taking the variety, we lose too much geometric information to hope