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AN ESTIMATE OF INFINITE CYCLIC COVERINGS AND KNOT THEORY

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In this paper we estimate the homology torsion module of an infinite cyclic covering space of an n-manifold by the homology of a Poincaré duality space of dimension $n-1$. To be concrete, we apply it to knot theory. In particular, it follows that any ribbon n-knot $K \subset S^{n+2}$ ($n \geqq 3$) is unknotted if $\pi_1(S^{n+2} - K) \cong \boldsymbol{Z}$. We add also in this paper a somewhat geometric proof to this unknotting criterion.

1. Statements of results. Let X be a compact, connected and smooth, piecewise-linear or topological n-manifold with nonzero 1st Betti number, i.e., $H^1(X; \boldsymbol{Z}) \neq 0$. Let $\tilde{X}$ be an infinite cyclic connected cover of X, that is, the cover of X associated with an indivisible element of $H^1(X; \boldsymbol{Z})$. We denote by $\langle t \rangle$ the covering transformation group of $\tilde{X}$ with a specified generator t. Let F be a field and $F\langle t \rangle$ be the group algebra of $\langle t \rangle$ over F. For $H_* = H_*(\tilde{X}; F)$ or $H_*(\tilde{X}, \partial\tilde{X}; F)$, H_* is canonically regarded as an $F\langle t \rangle$-module. We define $T_* = \mathrm{Tor}_{F\langle t \rangle} H_*$ and $T^* = \mathrm{Hom}_F[T_*, F]$. We assume $\tilde{X}$ is F-orientable. Note that $T_0(\tilde{X}; F) = H_0(\tilde{X}; F) \cong F$ and $T_{n-1}(\tilde{X}, \partial\tilde{X}; F) \cong F$. (Cf. [5, Duality Theorem (II) and Remark 1.3].) Let M be a connected Poincaré duality space with boundary ∂M of dimension $n-1$ over F.

THEOREM. *Suppose there is a map $f: (M, \partial M) \to (\tilde{X}, \partial\tilde{X})$ such that $f_* H_{n-1}(M, \partial M; F) = T_{n-1}(\tilde{X}, \partial\tilde{X}; F)$. Then*

$$\dim_F H_q(M; F) \geqq \dim_F T_q(\tilde{X}; F)$$

for all q. Further, if $f_* H_q(M; F) \subset T_q(\tilde{X}; F)$ for some q, then $f_* H_q(M; F) = T_q(\tilde{X}; F)$. In particular, if $T_q(\tilde{X}; F) = H_q(\tilde{X}; F)$ (e.g., $H_q(X; F) \cong H_q(S^1; F)$) *for some q, then the homomorphism*

$$f_*: H_q(M; F) \longrightarrow H_q(\tilde{X}; F)$$

is onto.

Note 1. Our proof will imply also that

$$\dim_F H_{n-q-1}(M, \partial M; F) \geqq \dim_F T_{n-q-1}(\tilde{X}, \partial\tilde{X}; F)$$

for all q and, if $f_* H_{n-q-1}(M, \partial M; F) \subset T_{n-q-1}(\tilde{X}, \partial\tilde{X}; F)$ for some q, then $f_* H_{n-q-1}(M, \partial M; F) = T_{n-q-1}(\tilde{X}, \partial\tilde{X}; F)$.

In case X is oriented and piecewise-linear and $\tilde{X}$ is obtained