PACIFIC JOURNAL OF MATHEMATICS
Vol. 90, No. 1, 1980

STRONG LIFTINGS COMMUTING WITH MINIMAL DISTAL FLOWS

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In this paper, we treat an aspect of the following problem. If a compact Hausdorff space X is given, and if T is a group of homeomorphisms of X which preserves a measure μ, then find conditions under which $M^\infty(X, \mu)$ admits a strong lifting (or strong linear lifting) which commutes with T. We will prove the following results.

Introduction. (1) Let (X, T) be a minimal distal flow. Then there exists an invariant measure μ such that $M^\infty(X, \mu)$ admits a strong linear lifting ρ commuting with T. The linear lifting ρ is "quasi-multiplicative" in the sense that $\rho(f \cdot g) = \rho(f) \cdot \rho(g)$ if $f \in C(X)$ and $g \in M^\infty(X, \mu)$. In particular, if (X, T) admits a unique invariant measure μ, then $M^\infty(X, \mu)$ admits ρ as above. This result may be viewed as a generalization of "Theorem LCG" of A. and C. Ionescu-Tulcea [7]; see 1.7. If T is *abelian*, then $M^\infty(X, \mu)$ admits a strong *lifting*.

(2) Let G be a compact group with Haar measure μ. Then $M^\infty(G, \mu)$ admits a strong linear lifting ρ (which is quasi-multiplicative), which commutes with both left and right multiplications on G.

The author would like to thank the referee for correcting and improving Corollary 3.10.

Preliminalies.

NOTATION 1.1. Let X be a compact Hausdorff space. If μ is a positive Radon measure on X, let $M^\infty(X, \mu)$ be the set of bounded, μ-measurable, complex-valued functions on X. Let $L^\infty(X, \mu)$ be the set of equivalence classes in $M^\infty(X, \mu)$ under the (usual) equivalence relation: $f \sim g \Leftrightarrow f - g = 0$ μ – a.e. If E is a Banach space, let $M^\infty(X, E, \mu) = \{f: X \to E | f$ is weakly μ-measurable, and Range (f) is precompact$\}$. (Recall $f: X \to E|$ is *weakly μ-measurable* if $x \to \langle f(x), e' \rangle$ is μ-measurable for all $e' = E' =$ topological dual of E.)

DEFINITIONS 1.2. Let X, μ be as in 1.1. A map ρ of $M^\infty(M, \mu)$ to itself is a *linear lifting* of $M^\infty(X, \mu)$ if: (i) $\rho(f) = f$ μ – a.e.; (ii) $f = g$ μ – a.e. $\Rightarrow \rho(f) = \rho(g)$ everywhere; (iii) $\rho(1) = 1$; (iv) $f \geqq 0 \Rightarrow \rho(f) \geqq 0$; (v) $\rho(af + bg) = a\rho(f) + b\rho(g)$ $(f, g \in M^\infty(X, \mu)$; $a, b \in C)$. If, in addition, (vi) $\rho(f \cdot g) = \rho(f) \cdot \rho(g)$ for all $f, g \in M^\infty(M, \mu)$, then ρ