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THE GALOIS GROUP OF A POLYNOMIAL WITH TWO INDETERMINATE COEFFICIENTS

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Suppose that $f(x) = \sum_{i=0}^{n} \alpha_i X^i (\alpha_0 \alpha_n \neq 0)$ is a polynomial in which two of the coefficients are indeterminates t, u and the remainder belong to a field F. We find the galois group of f over $F(t, u)$. In particular, it is the full symmetric group S_n provided that (as is obviously necessary) $f(X) \neq f_1(X^r)$ for any $r > 1$. The results are always valid if F has characteristic zero and hold under mild conditions involving the characteristic of F otherwise. Work of Uchida [10] and Smith [9] is extended even in the case of trinomials $X^n + tX^a + u$ on which they concentrated.

1. Introduction. Let F be any field and suppose that it has characteristic p, where $p = 0$ or is a prime. In [9], J. H. Smith, extending work of K. Uchida [10], proved that, if n and a are *co-prime* positive integers with $n > a$, then the trinomial $X^n + tX^a + u$, where t and u are independent indeterminates, has galois group S_n over $F(t, u)$, a proviso being that, if $p > 0$, then $p \nmid na(n - a)$. (Note, however, that this conveys no information whenever $p = 2$, for example.) Smith also conjectured that, subject to appropriate restriction involving the characteristic, the following holds. Let I be a subset (including 0) of the set $\{0, 1, \cdots, n - 1\}$ having cardinality at least 2 and such that the members of I together with n are co-prime. Let $T = \{t_i, i \in I\}$ be a set of indeterminates. Then the polynomial $X^n + \sum_{i=0}^{n-1} t_i x^i$ has galois group S_n over $F(T)$.

In this paper, we shall confirm this conjecture under mild conditions involving $p(>0)$, thereby extending even the range of validity of the trinomial theorem. In fact, we also relax the other assumptions. Specifically, we allow some of the t_i to be fixed nonzero members of F and insist only that two members of T be indeterminates. Indeed, even if the co-prime condition is dispensed with, so that the galois group is definitely not S_n, we can still describe what that group actually is. On the other hand, if, in fact, more than two members of T are indeterminates, then the nature of our results ensures that, in general, the relevant galois group is deducible by specialization.

Accordingly, from now on, let I denote a subset of *co-prime* integers from $\{0, 1, \cdots, n\}$ containing 0 and n and having cardinality $\geqq 3$. Write