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ANALYTIC FUNCTIONS IN TUBES WHICH ARE REPRESENTABLE BY FOURIER-LAPLACE INTEGRALS

RICHARD D. CARMICHAEL AND ELMER K. HAYASHI

Spaces of analytic functions in tubes in C^n which generalize the Hardy H^p spaces are defined and studied. In addition Cauchy and Poisson integrals of distributions in $\mathscr{D}'_{L^p}$ are analyzed.

1. Introduction. Bochner ([1] and [2]) has defined the Hardy $H^2(T^C)$ spaces for tubes $T^C = \boldsymbol{R}^n + iC$ in $\boldsymbol{C}^n$ where $C \subset \boldsymbol{R}^n$ is an open convex cone. Stein and Weiss [11] have studied the $H^p(T^B)$ spaces for arbitrary $p > 0$ and with respect to tubes T^B, B being an open proper subset of $\boldsymbol{R}^n$ [11, pp. 90-91]. Vladimirov [12, §§ 25.3-25.4] has considered analytic functions in T^C, C being an open connected cone, which satisfy the growth [12, p. 224, (64)]. Vladimirov has stated [12, p. 227, lines 4-5] that the growth which defines the H^2 functions of Bochner is more restrictive than [12, p. 224, (64)]. We show in this paper that the H^2 growth is not more restrictive than [12, p. 224, (64)] by showing that the functions of Vladimirov are exactly the H^2 functions. However, Vladimirov's growth has led us to define new spaces of analytic functions in tubes which have growth estimates that are more general than that of the $H^p(T^B)$ spaces, and we analyze these new spaces in this paper. Further, we study Cauchy and Poisson integrals of distributions in $\mathscr{D}'_{L^p}$.

The n-dimensional notation in this paper is described in [7, p. 386]. The definitions of a cone in $\boldsymbol{R}^n$, projection of a cone pr(C), compact subcone, and dual cone $C^* = \{t \in \boldsymbol{R}^n: \langle t, y\rangle \geqq 0, y \in C\}$ of a cone C are given in [12, p. 218]. Terminology concerning distributions is that of Schwartz [10]. The support of a distribution or function g is denoted supp(g). Definitions, properties, and relevant topologies of the function spaces $\mathscr{S}$, $\mathscr{D}_{L^p}$, $\mathscr{B} = \mathscr{D}_{L^\infty}$, and $\dot{\mathscr{B}}$ and of the distribution spaces $\mathscr{S}'$ and $\mathscr{D}'_{L^p}$ are in [10]. The L^1 and $\mathscr{S}'$ Fourier and inverse Fourier transforms are defined in [7, pp. 387-388] and [10, p. 250], respectively. The limit in the mean Fourier and inverse Fourier transforms of functions in L^p, $1 < p \leqq 2$, and L^q, $(1/p) + (1/q) = 1$, are in [8] and [3]. $\mathscr{F}[\phi(t); x]$ $(\mathscr{F}^{-1}[\phi(x); t])$ denotes the Fourier (inverse Fourier) transform of a function in the relevant sense. If $V \in \mathscr{S}'$ we denote its Fourier (inverse Fourier) transform by $\mathscr{F}[V] = \hat{V}$ $(\mathscr{F}^{-1}[V])$. For $\phi \in L^p$, $1 < p \leqq 2$, the Parseval inequality is