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UNIQUE FACTORIZATION OF RATIONAL HOMOTOPY TYPES

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A nontrivial, simply connected, rational homotopy type is called *irreducible,* unless it is the product of two nontrivial rational homotopy types. In this paper previous results are extended by proving that every *finitary,* simply connected rational homotopy type having *positive weights* is representable as the product of a *unique* set of irreducible types. On the way to this unique factorization result, it is proven that (in the rational homotopy category) *retracts of positive weight types* again have positive weights.

In contrast to the (above) uniqueness result for the rational homotopy category, unique factorization fails simultaneously (i.e., with a single example) in three finer geometric contexts: the differentiable, topological and (integral) homotopy categories. This well-known noncancellation example is discussed in the introduction of [1].

The results in this paper were announced in [9].

Let $\mathscr{P}\mathscr{F}$ denote the set of all *finitary,* simply connected rational homotopy types having *positive weights.* The term "*finitary*" means that either the rational homotopy or the rational cohomology is a finite dimensional vector space over Q, the field of rational numbers. The term "*positive weight*" is defined in the next section.

$\mathscr{P}\mathscr{F}$ contains the types of many interesting spaces (cf. [3], [6], [10], [12], [14]), including all simply connected, *formal,* finitary types. In fact, it is not an entirely trivial task to construct an example of a space which does not have positive weights. One of the earliest and simplest examples of such a space was constructed by Mimura and Toda [13] as the cofibre of a certain element of $\pi_{11}(S^3 \vee CP(2))$.

Throughout this paper, a rational homotopy type will be identified by its "minimal algebra" (or "minimal model" $d: M \rightarrow M$ denoted simply as M), where M is a free, simply-connected, graded-commutative Q-algebra and d is a decomposable, graded-derivation of degree 1, which is a differential on M. A self-contained introduction to this view of rational homotopy theory may be found in [5], [7] or [11]; a demonstration of the usefulness of this view may be found in [15].

1. Rational homotopy types having positive weights.

This section describes several equivalent characterizations of the set of