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BOUNDARY VALUE PROBLEMS FOR PARTIAL FUNCTIONAL DIFFERENTIAL EQUATIONS

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Sufficient conditions are given to ensure the existence of solutions for the boundary value problem

$$(1)\qquad y(t) = T(t)\phi(0) + \int_0^t T(t-s)F(y_s)ds \quad 0 \leqq t \leqq b$$

$$(*)\qquad My_0 + Ny_b = \psi\ , \quad \psi \in C(=C([-r, 0]; B) \text{ by def.})\ .$$

It is assumed that $T(t)$, $t \geqq 0$, is a strongly continuous semigroup of bounded linear operators on the Banach space B and $T(t)$, $t \geqq 0$, has infinitesimal generator A. The function F is continuous from C to B and M and N are bounded linear operators defined on C.

Denote by C the Banach space of continuous functions from $[-r, 0]$ into the Banach space B, where for each $\varphi \in C$, $||\varphi||_C = \sup_{-r\leqq\theta\leqq 0} \sup ||\varphi(\theta)||$. Let A be the infinitesimal generator of a strongly continuous semigroup of linear operators $T(t)$, $t \geqq 0$ mapping B into B and satisfying $|T(t)| \leqq e^{\omega t}$ for some real ω. We let F be a nonlinear continuous function from C into B. If $y(t)$ is a continuous function from $[0, T]$ to B for some $T > 0$, define the element $y_t \in C$ by $y_t(\theta) = y(t + \theta)$. Throughout this paper the reference $y(t)$ is a solution of Equation (1) (*) will mean $y(t)$ satisfies Equation (1) and the boundary condition (*). The statement $y(\varphi)(t)$ is a solution of Equation (1) will mean $y(t)$ satisfies Equation (1) and the initial condition $y_0 = \varphi$. The notation Equation (1) without (*) will always denote the initial value problem.

In a recent paper [8] C. Travis and G. Webb have considered initial value problems for Equation (1). With F satisfying

$$(2)\qquad ||F(\varphi) - F(\bar{\varphi})|| \leqq L||\varphi - \bar{\varphi}||_C$$

for some $L > 0$ and φ, $\bar{\varphi} \in C$, Travis and Webb obtain the existence of unique solutions of Equation (1) for each $\varphi \in C$. In another paper W. E. Fitzgibbon [2] has shown that global solutions of Equation (1) exist if F satisfies for each $\varphi \in C$

$$(3)\qquad ||F(\varphi)|| \leqq K_1||\varphi||_C + K_2 \quad \text{for some} \quad K_1, K_2 \in R\ ,$$

and if $T(t)$, $t > 0$ is compact.

When Equation (1) has unique solutions for each $\varphi \in C$, the mapping $U(t)\varphi = y_t(\varphi)$ is well defined for each $t \geqq 0$ and $\varphi \in C$. Here $y_t(\varphi)$ represents the element of C such that $y(\varphi)(t)$ is a solution of