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EMBEDDING ASYMPTOTICALLY STABLE DYNAMICAL SYSTEMS INTO RADIAL FLOWS IN l_2

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A dynamical system Π on a separable metric space, which has a globally asymptotically stable critical point p, can be embedded into a radial flow ρ on l_2 if and only if p is uniformaly asymptotically stable. Moreover, if Π can be embedded into ρ, then there is a locally compact subset Y of l_2 such that Π can be embedded into ρ restricted to Y.

In [1] the author showed that a dynamical system on a locally compact phase space, which has a globally asymptically stable critical point, can be embedded into the radial flow on l_2 defined by $z\rho t = c^t z$. Here we generalize this result and show that a dynamical system Π on a separable metric space which has a globally asymptotically stable critical point p, can be embedded into the radial flow ρ on l_2 if and only if p is uniformly asymptotically stable. Moreover, if Π can be embedded into ρ, then there is a locally compact subset Y of l_2 such that Π can be embedded into ρ restricted to Y.

A dynamical system on a topological space X is a continuous mapping $\Pi: X \times R \to X$ such that (where $x\Pi t = \Pi(x, t)$)

(1) $x\Pi 0 = x$ for every $x \in X$,

(2) $(x\Pi t)\Pi s = x\Pi(t + s)$ for every $x \in X$ and $s, t \in R$.

For $A \subset X$ and $B \subset R$, $A\Pi B$ will denote the set $\{x\Pi t: x \in A, t \in B\}$. In the special case $B = R$ we will write $C(A)$ instead of $A\Pi R$. An element $x \in X$ is called a *critical point* of Π if $C(x) = \{x\}$. A subset A of X is *invariant* if $C(A) = A$. We will let R^+ denote the nonnegative reals.

A compact subset M of X is called *stable* if for any neighborhood U of M there is a neighborhood V of M such that $V\Pi R^+ \subset U$. A stable subset M of X is called

(i) *asymptotically stable* if for any neighborhood U of M and any $x \in X$, there is a $T \in R$ such that $x\Pi[T, \infty) \subset U$,

(ii) *locally uniformly asymptotically stable* if for any $x \in X - M$, there is a neighborhood V of x such that for any neighborhood U of M there exists $T \in R$ such that $V\Pi[T, \infty) \subset U$.

(iii) *uniformly asymptotically stable* if there is a neighborhood U of M such that for any neighborhood $V \subset U$ of M there exists $T \in R$ such that $U\Pi[T, \infty) \subset V$.

A continuous function $L: R \to R^+$ is called a *Liapunov function* for a subset M of X if

(i) $L(x) = 0$ if and only if $x \in M$,