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ON THE DECOMPOSITION OF STATES OF SOME *-ALGEBRAS

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We study the direct integral decomposition of Op^*-algebras defined on a metrizable, dense domain of a separable Hilbert space. Applications to the decomposition into irreducible representations and into extremal states of representations and states of *-algebras with a countable dominating subset are given.

0. **Introduction.** This paper is concerned with the extension of the reduction theory for bounded operators [2] to algebras of unbounded operators and, as an application, with the decomposition of representations and states of *-algebras.

In a previous paper [3] we gave a meaning to the direct integral decomposition of unbounded operators, by considering them as bounded operators between several Hilbert spaces. Considering then, families $\mathscr{A}$ of unbounded operators defined on a *common* dense domain $\mathscr{D}$ of a separable Hilbert space $\mathscr{H}$ (the so-called Op^*-algebras [8]) and considering the decomposition $\mathscr{H} = \int_{\Lambda} \mathscr{H}(\lambda) d\mu(\lambda)$ of the Hilbert space with respect to an Abelian von Neumann algebra $\mathscr{M}$ contained in the strong commutant of $\mathscr{A}$, we have been able to define for almost every $\lambda \in \Lambda$, a domain $\mathscr{D}(\lambda)$ in $\mathscr{H}(\lambda)$ such that any $A \in \mathscr{A}$ can be written as $A = \int_{\Lambda} A(\lambda) d\mu(\lambda)$ where $A(\lambda)$ is a continuous operator from $\mathscr{D}(\lambda)$ into itself. Then, to any countable subalgebra $\mathscr{A}_0 \subset \mathscr{A}$ corresponds for almost every $\lambda \in \Lambda$ a countable Op^*-algebra $\mathscr{A}_0(\lambda)$ on $\mathscr{D}(\lambda)$.

However, in order to be sure that the domain $\mathscr{D}(\lambda)$ is nonzero and even dense in $\mathscr{H}(\lambda)$, almost everywhere, we had to ask that the $\mathscr{A}$-graph topology on $\mathscr{D}$ is metrizable and this assumption characterizes the class of Op^*-algebras we want to consider. For examples of such Op^*-algebras we refer to [13], [14].

In this paper, we extend the decomposition to uncountable Op^*-algebras. If we look in [2] how it is possible to get the reduction of a von Neumann algebra once one is able to decompose a single operator or a countable set of operators, one sees that the important things are the following:

(a) a von Neumann algebra is separable in the strong operator topology.

(b) this strong operator topology is metrizable.

(c) If $A^i \rightarrow A$ is this topology any if $\{A^i\}$ and A are decom-