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VECTOR VALUED DISTRIBUTIONS HAVING A SMOOTH CONVOLUTION INVERSE

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Let E, X be complex Banach spaces, (E, X) the space of linear operators from E into X equipped with its usual norm. We denote by $\mathscr{D}'(E)$ the space of E-valued distributions defined in $-\infty<t<\infty$ and by $\mathscr{D}_0(E)$ the subspace thereof consisting of distributions with support in $t\geq 0$. A distribution $P\in\mathscr{D}'_0((X; E))$ is said to have a *convolution inverse* (in symbols, $P\in\mathscr{D}'_0((E; X))^{-1}$ or simply $P\in\mathscr{D}_0'^{-1}$) if there exists $S\in\mathscr{D}'_0((E; X))$ such that

$$P*S=\delta\otimes I,\quad S*P=\delta\otimes J \tag{1}$$

where δ is the Dirac measure and I (resp. J) denotes the identity operator in E (resp. X). We examine the problem of characterizing those P which possess a convolution inverse $S=P^{-1}$ being smooth in various senses: infinitely differentiable, in a quasi-analytic class, analytic, etc.

1. Introduction. This paper continues the investigations in [6] on convolution inverses of vector valued distributions: for the necessary definitions and results see [14] or [15]. In particular, we use the definition of convolution in [15] as follows: if E, F, G, are Banach spaces and $U\in\mathscr{D}'_0((F; G))$, $V\in\mathscr{D}'_0((E; F))$ we understand by $U*V$ not the convolution in [15], which takes values in $(F; G)\hat{\otimes}(E; F)$ but rather its "composition" (in the sense of [15]) with the linear map from $(F; \mathrm{G})\hat{\otimes}(E; F)\to(E; G)$ induced by the (bilinear) product map from $(F, G)\times(E; F)$ into $(E; G)$; when U (resp. V) is, say a continuous $(F; G)$-valued (resp. $(E; F)$-valued) function, this definition coincides with the usual one. Necessary and sufficient conditions for a distribution $P\in\mathscr{D}'_0((X; E))$ to belong to $\mathscr{D}_0'^{-1}$ have been given in [6]. When $P\in\mathscr{S}'_0(X; E))$ (the space of all tempered, $(X; E)$-valued distributions with support in $t\geqq 0$) the Laplace transform $\mathfrak{P}(\lambda)=\mathscr{L}P(\lambda)=P(\exp(-\lambda(\cdot)))$ exists in the right half-plane, is analytic there, and grows no more than a polynomial as $|\lambda|\to\infty$. Denote by $\pi(P)$ the largest connected subset of the complex plane (containing the right half-plane) to which $\mathfrak{P}(\lambda)^{-1}$ can be extended as an analytic function. It follows from analyticity of $\mathfrak{P}(\lambda)$ that $\mathfrak{P}(\lambda)^{-1}$ exists in an open set $\rho(P)\subseteqq\pi(P)$ (perhaps empty) called the resolvent set of P and is analytic there. We have

THEOREM 1.1. *Let $P\in\mathscr{S}'_0((X; E))$. Then $P\in\mathscr{D}_0'^{-1}$ if and only*