

BOUNDS FOR THE EIGENFUNCTIONS OF A
TWO-PARAMETER SYSTEM OF ORDINARY
DIFFERENTIAL EQUATIONS
OF THE SECOND ORDER

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In previous papers the author has shown that, in contrast to the one-parameter case, the normalized eigenfunctions of two simultaneous Sturm-Liouville systems in two parameters are not necessarily uniformly bounded. Moreover, best possible bounds for the normalized eigenfunctions were also derived. However these results were only established under the assumption that the coefficients of our differential equations satisfied certain special conditions. Hence, in order to deal with problems which often arise in physical practice, it is important to extend our results to the case where the coefficients of our differential equations satisfy more general conditions than hitherto supposed. Accordingly, it is the object of this paper to derive best possible bounds for the normalized eigenfunctions of the simultaneous two-parameter systems in question under much weaker restrictions on their coefficients than was previously assumed.

The study of the behavior of the eigenvalues and eigenfunctions of multiparameter Sturm-Liouville systems was initiated by F. V. Atkinson [1, §4] who pointed out that the theory related to this subject was still far from clear. Since the appearance of Atkinson's paper, the author has obtained some relevant results, the most important of which are contained in papers [4] and [5] mentioned above. The results given in this paper therefore constitute a further stage in the development of the theory related to this subject.

Finally we mention that the methods used in this paper are quite different from those used in [4] and [5] wherein techniques from transition point theory and the method of asymptotic integration were employed. Here our results are established by utilizing the information given in [4] and [5], by making a through study of the general character of the solutions of our differential equations, and lastly, by making use of a Sobolev type inequality.

1. The main theorem. We shall be concerned here with the simultaneous two-parameter systems

$$(1) \quad \begin{aligned} y_1'' + (\lambda A_1(x_1) - \mu B_1(x_1) + q_1(x_1))y_1 &= 0, \\ 0 \leq x_1 \leq 1, \quad ' &= d/dx_1, \end{aligned}$$