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COMPLETIONS OF NOETHERIAN HEREDITARY PRIME RINGS

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If R is a Noetherian hereditary prime ring with Jacobson radical $J \neq (0)$ then, it has been shown that the J-adic completion of R is a Noetherian hereditary semi-prime ring; it is prime if and only if the maximal ideals of R form a single cycle. Among other things, one also finds the result: If R is a right Noetherian semi-local ring with $\bigcap_{n=1}^{\infty} J^n = (0)$ then, R is J-adic complete if and only if it is right linearly compact and J has the right AR-property.

As is clear from the literature, it is not known whether $\hat{R}$, the J-adic completion of a Noetherian hereditary prime ring R with $J \neq (0)$, is (right) Noetherian. It has been shown here that $\hat{R}$ is Noetherian. The approach taken proves not only this but also that $\hat{R}$ is hereditary and semi-prime. In view of Michler's structure theorem for semi-local Noetherian hereditary prime rings [13; 6.5], the structure of $\hat{R}$ then is completely determined since, in the present case, $\hat{R}$ decomposes into prime rings each complete in its radical topology. Thus the theorem generalizes the corresponding result for Dedekind prime rings obtained in [6]. (It is immediate that if R is a Noetherian semi-local ring which is either hereditary or serial then $\hat{R}$ is Noetherian, and respectively hereditary or serial.) Section 3 is concerned with the above result on HNP-rings (Hereditary Noetherian prime rings), and § 4 gives a necessary and sufficient condition so that $\hat{R}$ be an HNP-ring. Section 1 gives preliminaries. Section 2 contains some general results needed for the main theorem; in particular, it contains some properties of the endomorphism ring of a quasi-injective module over a semi-local ring satisfying some additional conditions.

1. Preliminaries. All rings considered have unity, need not be commutative and the modules are unitary. J denotes the Jacobson radical of a ring R. R is said to be *semi-local* if R/J is Artinian. An ideal I of a ring has the right AR-property if for each right ideal B there is an n such that $B \cap I^n \subset BI$. The socle of a module M is denoted by soc$M$. The socle series of a module M is the ascending sequence $\{\mathrm{soc}_n M: n \geqq 0\}$ of submodules of M defined as follows: $\mathrm{soc}_0 M = (0)$; $\mathrm{soc}_{n+1} M = \pi_n^{-1}(\mathrm{soc}(M/\mathrm{soc}_n M))$ where $\pi_n: M \rightarrow M/\mathrm{soc}_n M$ is the canonical map. If M is a right R-module and if T and N are subsets of R and M respectively, then $\mathrm{ann}_R N$ and