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TWO APPLICATIONS OF THE SCHUR-NEVANLINNA ALGORITHM

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Two applications of the Schur-Nevanlinna algorithm are given. The first application gives new information about the Nevanlinna-Pick interpolation problem. The second application concerns the constructive approximation to bounded measurable functions on the unit circle by functions from H^{∞}.

1. Introduction. The Schur-Nevanlinna algorithm was developed by I. Schur [7] and refined by R. Nevanilnna [6] in the study of certain interpolation problems for bounded analytic functions.

The main idea in the first application is to combine Nevanlinna's algorithm with a certain uniqueness criterion due to Denjoy. This gives new information about solutions of the Nevanlinna-Pick interpolation problem.

The second application concerns the constructive approximation to bounded measurable functions on the unit circle $T = \{z: |z| \leqq 1\}$, by functions from H^{∞}. As usual, H^{∞} consists of the bounded analytic functions in the unit disc $D = \{z: |z| < 1\}$. They are extended to $D \cup T$ by taking radial limits, thanks to a well known theorem of Fatou. The main tool here is Schur's algorithm [7]. Assuming the recent result about duality between H^1 and BMO [1], our method also yields a constructive decomposition of functions f in the class BMO (functions of bounded mean oscillation). This has recently been done by P. Jones [4], using entirely different methods.

2. Nevanlinna's algorithm and Denjoy's criterion. It will be necessary to describe the results and ideas in Nevanlinna's fundamental paper [6], in some detail.

The space H^{∞} introduced above, is a Banach space with the norm $\|f\| = \sup\{|f(z)|, z \in D\}$. Let $\{z_\nu\}$ be a sequence of distinct points in D, and consider the interpolation problem

$$(*) \qquad w(z_\nu) = w_\nu\,, \quad \nu = 1, 2, \cdots$$

where $\{w_\nu\}$ is a specified sequence of complex numbers, and $w \in H^{\infty}$, $\|w\| \leqq 1$.

We assume that (*) has at least two solutions. Then R. Nevanlinna [6] has shown that all solutions to (*) are given by the following formula