PACIFIC JOURNAL OF MATHEMATICS
Vol. 91, No. 1, 1980

ON DETERMINING REGULAR BEHAVIOR FROM THE RECURRENCE FORMULA FOR ORTHOGONAL POLYNOMIALS

DANIEL P. MAKI

Ullman and Erdös and Freud have studied the distribution of the zeros of certain classes of orthogonal polynomials. Among other results they have shown that for a wide class of weight functions the associated orthogonal polynomials all have the same limiting zero distribution. We show, in a related result, that in certain cases one can deduce the limiting distribution of the zeros of the orthogonal polynomials without explicitly knowing the weight function (or the distribution function) of the orthogonal polynomials. In particular, for polynomials with certain types of triple recurrence formula we show that the limiting zero distribution is always the one studied by Ullman and Erdös and Freud. Polynomials with this limiting zero distribution are said to have "regular zero behavior".

In § 2 we give the basic definitions and notation needed for our result. Our main theorem is in § 3, and § 4 contains some related comments and examples.

2. Definition and notation. We adopt the notation of Erdös and Freud in [3].

Let $d\alpha(x)$ be a nonnegative measure on $(-\infty, \infty)$ for which all moments

$$\mu_n(d\alpha) = \int_{-\infty}^{+\infty} x^m d\alpha(x)$$

exist and are finite and $\mu_0(d\alpha) = 1$. We consider the orthonormal polynomials

$$p_n(d\alpha; x) = \gamma_n(d\alpha) \prod_{k=1}^{n} [x - x_{kn}(d\alpha)] \tag{2.1}$$

which satisfy $\gamma_n(d\alpha) > 0$ and $\int_{-\infty}^{+\infty} p_n p_m d\alpha = \delta_{nm}$, where δ_{nm} is the Kronecker delta. We are also interested in the monic polynomials $\{q_n(x)\}$ defined by $q_n(x) = p_n(d\alpha; x)/\gamma_n(d\alpha)$, $n \geqq 0$. The polynomials $\{q_n(x)\}$ are orthogonal with respect to $d\alpha(x)$, and thus they obey a triple recurrence formula of the following form (see [6]):

$$\begin{aligned} &q_n(x) = (x - c_n)q_{n-1} - \lambda_n q_{n-2} \\ &q_{-1}(x) \equiv 0 , \qquad q_0(x) \equiv 1 \\ &c_n \text{ real}, \ \lambda_{n+1} > 0 , \qquad n = 1, 2, 3, \cdots . \end{aligned} \tag{2.2}$$