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ANALYTIC *H*-SPACES, CAMPBELL-HAUSDORFF FORMULA, AND ALTERNATIVE ALGEBRAS

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Analytic H-spaces are shown to be local analytic loops (satisfying the cancellation laws). Then power associative local analytic loops are investigated and these are shown to be exactly the class to which a local loop belongs if there is a choice of coordinate system, f, for which the multiplication obeys $V(sx, tx) = sx + tx$. Here x is near 0 in R^n, each of the numbers s, t and $s + t$ is in $[0, 1]$ and V is the pulldown of the local loop multiplication via f. Homomorphism of such local loops are investigated and the set of such automorphism is shown to be isomorphic to a certain group of linear maps. Also generalizing the Lie group-Lie algebra situation, certain anti-commutative algebras are introduced to study these local loops. Finally these results are applied to local loops whose multiplication is induced by a power associative algebra. A Campbell-Hausdorff formula is shown to hold when the algebra is alternative and is related to the inverse property in the local loop. A relationship between S^7 and simple Malcev algebras is given.

Introduction. As given in [15], an H-space is a set M with multiplication function $m: M \times M \to M$ having an identity element e. As a variation of this and local groups, the triple (M, E, m) is said to be a local analytic H-space provided M is an analytic manifold, E is an open set of M containing e, and m is an analytic function from $E \times E$ to M satisfying $m(e, x) = m(x, e) = x$ for each $x \in E$. We show in § 1 that these local analytic H-spaces satisfy the two-sided cancellation laws locally so that they are actually local loops and inverses exist locally.

Suppose (M, E, m) is a local analytic H-space and x is in E. Let $x^0 = e$ and if x^{n-1} is in E, let $x^n = m(x, x^{n-1})$. Then (M, E, m) is power associative if and only if for positive integers m, n

$$m(x^n, x^m) = x^{n+m}$$

whenever each of x^n and x^m is in E and x^{m+n} exists. Power associative analytic loops include Lie groups as well as seven-sphere S^7 with multiplication induced from the Cayley numbers.

In describing the structure of analytic local H-spaces it is convenient to choose a coordinate system f with domain a neighborhood of e so that $f(e) = 0$. There is then a neighborhood D of 0 in R^n so that the equation

$$V(f(x), f(y)) = f(m(x, y))$$